PRINT Last name:____________ First name:____________

Signature:____________ Student ID:____________

Math 152 Exam 3, Fall 2005

Instructions. For the multiple choice problems, there is no partial credit. For the work-out problems, you must show the mathematical steps of the solution in order to receive full credit; writing only the answer will receive no credit. Written explanations for each step are not required (due to time constraint) but an incorrect solution with a correct written explanation might receive partial credit. You do not have to verify the preconditions of a convergence test unless it is explicitly requested.

Conditions. No calculators, computers, notes, books, or scratch paper. By writing your name on the exam you certify that all work is your own, under penalty of all remedies outlined in the IIT student rules. **Please do not talk until you are away from the room.**

Time limit: 50 minutes (strict).

NOTE: The topics may not be in order either of increasing difficulty or of the order they were covered in the course.

POSSIBLY USEFUL FORMULAS

$\sec^2 x = \tan^2 x + 1$

$M_n = \Delta x \left[f\left(\frac{x_0+x_1}{2}\right) + f\left(\frac{x_1+x_2}{2}\right) + \cdots + f\left(\frac{x_{n-1}+x_n}{2}\right)\right]$

$\cos^2 x = \frac{1+\cos 2x}{2}$

$T_n = \frac{\Delta x}{2}\left[f(x_0) + 2f(x_1) + \cdots + 2f(x_{n-1}) + f(x_n)\right]$

$\int \frac{dx}{1+x^2} = \tan^{-1} x + C$

$\int \tan x\, dx = -\ln|\cos x| + C$

$PV = nRT$

$|E_M| < \frac{K(b-a)^3}{24n^2} \quad (K \geq f''(x))$

$F = \rho g A d$

$|E_T| < \frac{K(b-a)^3}{12n^2} \quad (K \geq f''(x))$

$|R_n(x)| \leq \frac{M}{(n+1)!}|x-a|^{n+1}$

$\mathbf{n}\cdot(\mathbf{r}-\mathbf{r}_0) = 0$

$S_n = \frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\right]$

$|E_S| < \frac{K(b-a)^5}{180n^4} \quad (K \geq f^{(4)}(x))$

$\int_{n+1}^{\infty} f(x)\,dx \leq R_n \leq \int_n^{\infty} f(x)\,dx$

$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + C$

$\frac{1}{1+x^2} = \sum_{n=0}^{\infty}(-1)^n x^{2n}$

$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$

$|s - s_n| \leq |a_{n+1}|$

$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$

$\sin 2x = 2\sin x\cos x$

$\text{Vol} = \int_a^b 2\pi\left[(f(x))^2 - (g(x))^2\right]\,dx$

$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$

Part I. (10pts ea.) MULTIPLE CHOICE–NO PARTIAL CREDIT NO CALCULATORS

1. Compute the limit $\displaystyle\lim_{n\to\infty} \frac{\sqrt[4]{n^5 - 3n}}{\sqrt[4]{81n^5 + 4n^4 - 3n}}$.

   a) $\frac{1}{27}$ b) $\frac{1}{9}$ c) $\frac{1}{3}$ d) $\frac{1}{4}$ e) 0

2. Which statement is true about the series $\displaystyle\sum_{n=1}^{\infty} \ln\left(\frac{n}{n+1}\right)$?

   a) It is a convergent p-series whose n^{th} partial sum is $s_n = -\ln(n+1)$.

   b) It is a divergent telescoping series whose n^{th} partial sum is $s_n = -\ln(n+1)$.

   c) It is convergent because $\lim_{n\to\infty} \ln(n/(n+1)) = 0$.

   d) It is divergent because $\lim_{n\to\infty} n/(n+1) = 1$.

   e) It is convergent because $\ln(n/(n+1)) < 1/n$ and $\sum_{n=1}^{\infty} \frac{1}{n}$ is convergent.

3. The sum of the series $\dfrac{2}{3} + \dfrac{2}{3^2} + \dfrac{2}{3^3} + \dfrac{2}{3^4} + \cdots$ is

   a) ∞ b) 3 c) 2 d) 1 e) $\frac{1}{3}$

4. Select the correct completion of the statement: The series $\displaystyle\sum_{n=1}^{\infty} \frac{n^3}{n^4 - \frac{1}{2}}$

   a) converges by the ratio test.

   b) diverges by the ratio test.

   c) converges, by comparison with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^3}$.

   d) diverges, by comparison with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$.

   e) converges, by comparison with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2n}$.

5. Select the expression which represents the area bounded by one loop of the graph of the polar function $r = \sin(3\theta)$.

   a) $\displaystyle\int_{-\pi/6}^{\pi/6} \pi \sin^2(3\theta)\, d\theta$

   b) $\displaystyle\int_{-\pi/6}^{\pi/6} \sin(3\theta)\, d\theta$

   c) $\displaystyle\int_{0}^{2\pi/3} \sin(3\theta)\, d\theta$

   d) $\displaystyle\int_{0}^{2\pi/3} \frac{1}{2} \sin^2(3\theta)\, d\theta$

   e) $\displaystyle\int_{0}^{\pi/3} \frac{1}{2} \sin^2(3\theta)\, d\theta$

Part II. SHOW WORK FOR FULL CREDIT NO CALCULATORS

6. **(15 pts)** Use one of the integral remainder estimates to estimate the value of $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$ to within 2×10^{-4}. **Clearly identify** the estimate, error, and error bound. Show your work which guarantees that the estimate is within this amount of the actual sum.

7. **(15 pts)** Assume that $f(x) = \sin x$ has power series representation

$$\sin x \;=\; \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

with radius of convergence $R = \infty$. Find a power series representation for $g(x) = \cos x$ and determine, with justification, its radius of convergence.

8. **(20 pts)** Find the center, radius and the **interval** of convergence of the power series

$$\sum_{n=1}^{\infty}(-1)^n\frac{(x-3)^n}{n\cdot 5^n}\,.$$