PRINT Last name:________________ First name:________________

Signature:________________ Student ID:________________

Math 152 Exam 2, Fall 2005

Instructions. You must show the mathematical steps of the solution in order to receive full credit; writing only the answer will receive no credit. Written explanations for each step are not required (due to time constraint) but an incorrect solution with a correct written explanation might receive partial credit.

Conditions. No calculators, computers, notes, books, or scratch paper. By writing your name on the exam you certify that all work is your own, under penalty of all remedies outlined in the IIT student rules. **Please do not talk until you are away from the room.**

Time limit: 50 minutes (strict).

NOTE: The topics may not be in order either of increasing difficulty or of the order they were covered in the course.

POSSIBLY USEFUL FORMULAS

$\sec^2 x = \tan^2 x + 1$

$\cos^2 x = \frac{1+\cos 2x}{2}$

$\int \frac{dx}{1+x^2} = \tan^{-1} x + C$

$PV = nRT$

$F = \rho g A d$

$|R_n(x)| \leq \frac{M}{(n+1)!}|x-a|^{n+1}$

$M_n = \Delta x \left[f\left(\frac{x_0+x_1}{2}\right) + f\left(\frac{x_1+x_2}{2}\right) + \cdots + f\left(\frac{x_{n-1}+x_n}{2}\right)\right]$

$T_n = \frac{\Delta x}{2}\left[f(x_0) + 2f(x_1) + \cdots + 2f(x_{n-1}) + f(x_n)\right]$

$\int \tan x\, dx = -\ln|\cos x| + C$

$|E_M| < \frac{K(b-a)^3}{24n^2} \quad (K \geq f''(x))$

$|E_T| < \frac{K(b-a)^3}{12n^2} \quad (K \geq f''(x))$

$\mathbf{n} \cdot (\mathbf{r} - \mathbf{r}_0) = 0$

$S_n = \frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\right]$

$|E_S| < \frac{K(b-a)^5}{180n^4} \quad (K \geq f^{(4)}(x))$

$\int_{n+1}^{\infty} f(x)\, dx \leq s - s_n \leq \int_n^{\infty} f(x)\, dx$

$\int \frac{1}{\sqrt{1-x^2}}\, dx = \sin^{-1} x + C$

$\frac{1}{1+x^2} = \sum_{n=0}^{\infty} (-1)^n x^{2n}$

$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$

$\sin 2x = 2 \sin x \cos x$

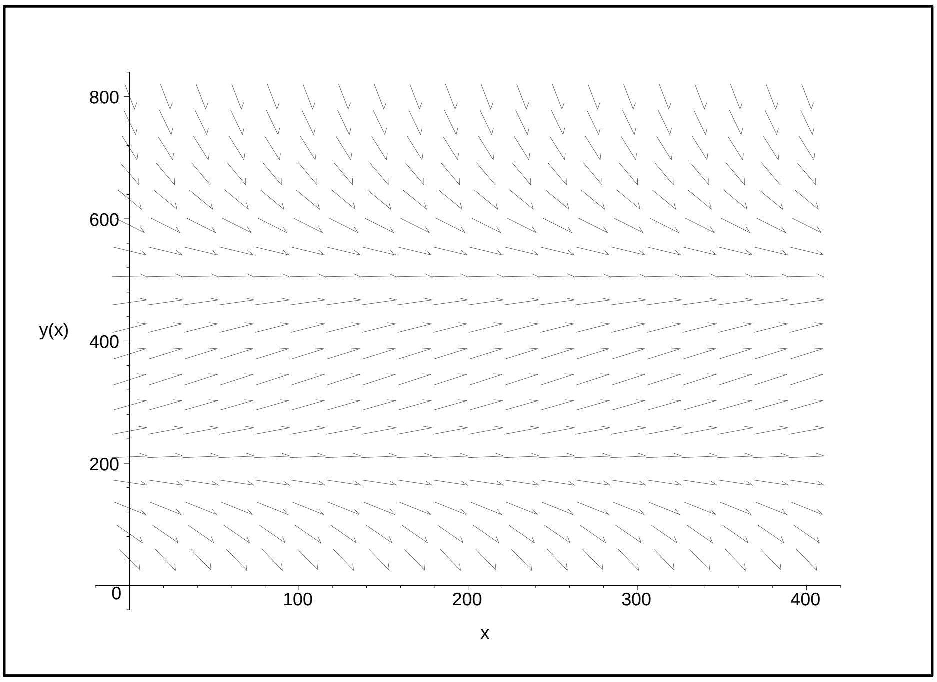
$\text{Vol} = \int_a^b 2\pi \left[(f(x))^2 - (g(x))^2\right]\, dx$

SHOW WORK FOR FULL CREDIT NO CALCULATORS

1. Find the partial fraction decomposition of $\dfrac{x^3 + x^2}{(x^2 + 1)^2}$.

2. Given the integral $\displaystyle\int_e^\infty \frac{1}{x(\ln x)^p}\,dx$,
   (a) Find a value of p so that the integral diverges.
   (b) Find a value of p so that the integral converges, and compute the integral in this case.
   (Hint for both parts: Find the *indefinite* integral only once using the substitution $u = \ln x$.)

3. Sketch four particular solutions to the differential equation whose slope field is below: **(a)** 2 equilibrium solutions, **(b)** 1 increasing solution, and **(c)** 1 decreasing solution.



4. Plutonium-238, a satellite power source, has a half-life of 88 years. Compute the time at which 10% of a sample of plutonium-238 remains. (Recall that the rate of decay of a radioactive substance is proportional to the amount remaining.)

5. For this problem, use the differential equation for Newton's Law of Cooling:

$$\frac{dT}{dt} = k(T - T_s)\,,$$

where T is the temperature, t is time, T_s is the ambient temperature, and k is a constant.

Question: Water at temperature 60^oF is placed in a freezer which is at 30^oF. After 1 hour, the water is at temperature 40^oF. How long does it take the water to freeze (i.e., at 32^oF)?

6. Find the general solution to the differential equation

$$x^2y' + 3xy = e^x\,,$$

where $x > 0$ (this means the general solution is defined only for $x > 0$).

7. The plot below is of the parametric equations

$$\begin{aligned} x &= t \cdot \cos(2t) \\ y &= t \cdot \sin(2t) \\ & 0 \ \le t \le \pi \, . \end{aligned}$$

(a) Find all values of t in the interval $(0, \pi)$ which correspond to horizontal or vertical lines. (Do not check $t = 0$ or $t = \pi$.)
(b) Label the corresponding horizontal and vertical tangent lines on the plot above with the appropriate values of t.