PRINT Last name:________________ First name:________________

Signature:________________ Student ID:________________

Math 152 Exam 1, Fall 2005

Instructions. *Part I* is multiple choice. There will be no partial credit. Clearly indicate your answer, especially if you change an answer. Problems with two indicated answers will receive no credit.
Part II is work-out problems. You must show work in order to receive full credit. Partial credit is possible for work which makes positive progress toward the solution.
Part III consists of the more conceptual problems; otherwise the instructions are the same as Part II.

Conditions. No calculators, notes, books, or scratch paper. By writing your name on the exam you certify that all work is your own, under penalty of all remedies outlined in the TAMU student rules. Please do not talk until after leaving the room.

Time limit: 2 hours (strict).

NOTE: The topics may not be in order either of increasing difficulty or of the order they were covered in the course.

POSSIBLY USEFUL FORMULAS

$\sec^2 x = \tan^2 x + 1$

$\cos^2 x = \frac{1+\cos 2x}{2}$

$\int \frac{dx}{1+x^2} = \tan^{-1} x + C$

$PV = nRT$

$F = \rho g A d$

$|R_n(x)| \le \frac{M}{(n+1)!}|x-a|^{n+1}$

$M_n = \Delta x \left[f\left(\frac{x_0+x_1}{2}\right) + f\left(\frac{x_1+x_2}{2}\right) + \cdots + f\left(\frac{x_{n-1}+x_n}{2}\right)\right]$

$T_n = \frac{\Delta x}{2}\left[f(x_0) + 2f(x_1) + \cdots + 2f(x_{n-1}) + f(x_n)\right]$

$\int \tan x\, dx = -\ln|\cos x| + C$

$|E_M| < \frac{K(b-a)^3}{24n^2} \quad (K \ge f''(x))$

$|E_T| < \frac{K(b-a)^3}{12n^2} \quad (K \ge f''(x))$

$\mathbf{n} \cdot (\mathbf{r} - \mathbf{r}_0) = 0$

$S_n = \frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\right]$

$|E_S| < \frac{K(b-a)^5}{180n^4} \quad (K \ge f^{(4)}(x))$

$\int_{n+1}^{\infty} f(x)\, dx \le s - s_n \le \int_n^{\infty} f(x)\, dx$

$\int \frac{1}{\sqrt{1-x^2}}\, dx = \sin^{-1} x + C$

$\frac{1}{1+x^2} = \sum_{n=0}^{\infty} (-1)^n x^{2n}$

$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$

$\sin 2x = 2\sin x \cos x$

$\text{Vol} = \int_a^b 2\pi\left[(f(x))^2 - (g(x))^2\right]\, dx$

SHOW WORK FOR FULL CREDIT NO CALCULATORS

1. Compute the derivative of $f(x) = \ln\left(\dfrac{x+3}{\sin(x^2)}\right)$

2. In a radioactive decay model, the amount of an isotope $A(t)$ in kilograms at time t years is given by

$$A(t) = A_0 e^{-(\ln 2)t/H},$$

where A_0 is the initial amount at $t = 0$ and H is a positive constant called the "half-life."

(a) Rearrange the right-hand side using properties of $y = \exp(x)$ and $y = \ln x$ so that the formula more clearly indicates why H is called the "half-life."

(b) Find the time t at which 10% of the original amount remains.

3. Find an exponential function of the form $f(x) = C \cdot a^x$ which satisfies $f'(2) = 75 \ln 5$ and $f(0) = 3$.

4. Differentiate the function $y = x^{\tan x}$.

5. Find the exact value of $\tan^{-1}(\tan(3\pi/4))$.

6. Evaluate the integral $\displaystyle\int \frac{x}{\sqrt{1-x^4}}\,dx$.

7. Evaluate the limit $\displaystyle\lim_{x\to 0} \frac{e^{4x} - 1 - 4x}{x^2}$.

8. Evaluate the limit $\displaystyle\lim_{x\to 0^+} \sin x \ln x$.

9. Compute the definite integral $\displaystyle\int_{-1}^{1} x \sin(\pi x)\, dx$.

10. Evaluate the indefinite integral $\displaystyle\int \tan^3(2x)\sec^5(2x)\,dx$.

11. Evaluate the indefinite integral $\displaystyle\int \frac{dx}{\sqrt{x^2+4}}$.

12. Compute the partial fraction decomposition of $\dfrac{10}{(x^2-1)(x^2+9)}$, being sure to leave the final answer in terms of x.