Homework 14: Poker with 5 suits (Due 10am 5/9/08)

Between 1-3 persons can submit the same worksheet. By signing your name you agree that you have met with the team and contributed constructively to solving every problem.

Staple separate paper with combinatorial justification for each probability. Work that is not easily readable will receive no credit.

Team Member 1 ____

Team Member 2 ____

Team Member 3 ____

For this worksheet, assume a deck of cards has 13 ranks and **5 suits** (13*5=65 total cards). As a set the deck is
{A,2,3,4,5,6,7,8,9,10,J,Q,K}x{C,D,H,S,B} (The new suit is B = Bells.)

In the game 5-card hands are still dealt. Now a new type of hand is possible – a **rainbow** hand. A rainbow hand is a 5-card hand in which there is one card from each suit (one club, one diamond, one heart, one spade, and one bell). Various types of hands are defined below. Compute the probability of getting each *type* of hand, assuming that each hand is equally likely. Tabulate these probabilities in the table, both in terms of combinatorial coefficients, and as decimal approximations to 3 places. Give the "winning order" of the hands by ordering them from lowest probability to highest probability. (Best hand = Royal Rainbow or Royal Flush?)

Definitions of hand types

<u>Royal Rainbow</u> 10,J,Q,K,A with all distinct suits

<u>Royal Flush</u> 10,J,Q,K,A all of same suit

<u>Straight Rainbow</u> All consecutive ranks and all distinct suits, but not a Royal Rainbow. A,2,3,4,5 and 9,10,J,Q,K are considered to be in consecutive rank order.

<u>Straight Flush</u> All consecutive ranks and all the same suit, but not a Royal Flush. A,2,3,4,5 all of the same suit is a straight flush.

<u>Straight</u> All consecutive ranks but not a royal rainbow, royal flush, straight rainbow, or straight flush. A,2,3,4,5 is a straight.

<u>Flush</u> All cards of same suit, but not a royal flush or straight flush.

<u>Rainbow 4 of a Kind</u> 4 cards of the same rank, and all 5 suits are present.

<u>4 of a Kind</u> 4 cards of the same rank, but not rainbow.

<u>Rainbow Full House</u> 3 cards of one rank and 2 cards of a second rank, and all 5 suits present.

<u>Full House</u> 3 cards of one rank and 2 cards of a second rank, but not rainbow

<u>Rainbow 3 of a Kind</u> 3 cards of one rank, and all 5 suits present, but not a rainbow 4 of a kind or rainbow full house.

<u>3 of a Kind</u> 3 cards of one rank, but not 4 of a kind or a full house, and not rainbow.

<u>Rainbow 2 Pair</u> Two cards of one rank and Two cards of a second rank, and all 5 suits present, but not a rainbow full house.

<u>2 Pair</u> Two cards of one rank and Two cards of a second rank, but not a full house and not rainbow.

<u>Rainbow Pair</u> 2 cards of one rank and all 5 suits present, but not a rainbow 4 of a kind, full house, 3 of a kind, or 2 pair.

<u>1 Pair</u>. 2 cards of one rank but not rainbow, 4 of a kind, full house, 3 of a kind, or 2 pair.

<u>Rainbow</u> All cards have distinct suits, but not a royal rainbow, straight rainbow, rainbow 4 of a kind, rainbow full house, rainbow 3 of a kind, rainbow 2 pair, or rainbow pair

Hand Type	Probability	3-decimal approximation	Winning Order (1^{st}-17^{th})
5 of a Kind (Rainbow)	Select rank: 13 ways All 5 suits of this rank must be present: 1 way **Total: 13 ways Probability: 13/C(65,5)**	.00000157	**2**
Royal Rainbow	1 each of ranks 10, J, Q, K, A must be present. Select suit of 10: 5 ways Select suit of J from remaining suits: 4 ways Select suit of Q from remaining suits: 3 ways Select suit of K from remaining suits: 2 ways Select suit of A from remaining suits: 1 way **Total: 5!=120 ways Probability: 120/C(65,5)**	.0000145	**4**
Royal Flush	Select suit: 5 ways All 5 ranks 10, J, Q, K, A must be present. **Total: 5 ways Probability: 5/C(65,5)**	.000000605	**1**
Straight Rainbow	Select Lowest rank in Straight (not 10): 9 ways Select suit of lowest rank: 5 ways Select suit of 2nd rank card from other suits: 4 ways Select suit of 3rd rank card from other suits: 3 ways Select suit of 4th rank card from other suits: 2 ways Select suit of 5th rank card from other suits: 1 way **Total: 9*5!=1080 ways Probability: 1080/C(65,5)**	.000131	**6**
Straight Flush	Select lowest rank in straight (not 10): 9 ways Select suit of straight: 5 ways **Total: 45 ways Probability: 45/C(65,5)**	.00000545	**3**
Straight	Count all ways of 5 consecutive cards, then subtract Straight Flush, Straight Rainbow, Royal Flush, Royal Rainbow. Select lowest rank of consecutive cards: 10 ways Select suit of each card: 5^5 ways **subtotal: 10*5^5 ways** Subtract Straight Flush: -45 Subtract Rainbow: -9*5! Subtract Royal Flush: -5 Subtract Royal Rainbow: -5! **Total: 10*5^5-50-10*5!=30,000 ways Probability: 30000/C(65,5)**	.00363	**13**
Flush	Count all ways to choose all same suit, then subtract Straight Flush and Royal Flush Select suit: 5 ways Select 5 ranks from this suit: C(13,5) ways **subtotal: 5*C(13,5) ways** Subtract Straight Flush: -45 Subtract Royal Flush: -5 **Total: 5*C(13,5)-50=6385 ways Probability: 6385/C(65,5)**	.000773	**9**
Rainbow 4 of a Kind	Select rank of 4 of a kind: 13 ways Select rank of other card: 12 ways Select suits of the 4 of a kind: C(5,4) Select 5^{th} suit for other card: 1 way	.0000944	**5**

	Total 13*12*C(5,4)=780 ways Probability: 780/C(65,5)		
4 of a Kind	Select rank of 4 of a kind : 13 ways Select rank of other card: 12 ways Select suits of the 4 of a kind: C(5,4) Select repeat suit for other card: 4 ways **Total 13*12*C(5,4)*4=3120 ways Probability: 3120/C(65,5)**	.000378	**8**
Rainbow Full House	Select rank of triple: 13 ways Select rank of pair: 12 ways Select 3 suits for triple: C(5,3) ways (This determines the suits for the pair) **Total 13*12*C(5,3)=1560 ways Probability: 1560/C(65,5)**	.000189	**7**
Full House	Start with a general Full House and subtract Rainbow Full House Select rank of triple: 13 ways Select rank of pair: 12 ways Select 3 suits for triple: C(5,3) ways Select 2 suits for pair: C(5,2) ways **subtotal: 13*12*C(5,3)*C(5,2) ways** Subtract Rainbow Full House: -13*12*C(5,3) **Total 13*12*C(5,3)(C(5,2)-1)=14040 ways Probability: 14040/C(65,5)**	.00170	**10**
Rainbow 3 of a Kind	Select rank of triple: 13 ways Select 2 distinct ranks for the other cards: C(12,2) ways Select 3 suits for triple: C(5,3) ways Assign other 2 suits to other 2 cards: 2 ways **Total 13*C(12,2)*C(5,3)*2=17160 ways Probability: 17160/C(65,5)**	.00208	**11**
3 of a Kind	Start with general 3 of a kind and subtract Rainbow 3 of a Kind Select rank of triple: 13 ways Select 2 distinct ranks for the other cards: C(12,2) ways Select 3 suits for triple: C(5,3) ways Select 2 suits for other 2 cards: 5^2 ways **subtotal: 13*C(5,2)*C(5,3)*5^2 ways** Subtract Rainbow 3 of a Kind: -17160 **Total 13*C(12,2)*C(5,3)*(5^2-2)=197340 ways Probability: 197340/C(65,5)**	.0239	**16**
Rainbow 2 Pair	Select 2 ranks for the 2 pair: C(13,2) ways Select 2 suits for higher rank pair: C(5,2) ways Select 2 remaining suits for lower rank pair: C(3,2) ways Select singleton rank: 11 ways Suit of singleton rank is determined **Total C(13,2)*C(5,2)*C(3,2)*11=25740 ways Probability: 25740/C(65,5)**	.00312	**12**
2 Pair	Start with general 2 Pair and subtract Rainbow 2 Pair Select 2 ranks for the 2 pair: C(13,2) ways Select 2 suits for higher rank pair: C(5,2) ways	.0488	**17**

	Select 2 suits for lower rank pair: C(5,2) ways Select rank for singleton: 11 ways Select suit for singleton: 5 ways **subtotal: C(13,2)*C(5,2)*C(5,2)*11*5** Subtract Rainbow 2 Pair: -C(13,2)*C(5,2)*C(3,2)*11 **Total C(13,2)*C(5,2)*11*(C(5,2)*5-C(3,2))=403260 ways Probability: 403260/C(65,5)**		
Rainbow Pair	Select rank for the pair: 13 ways Select 3 ranks for singletons: C(12,3) ways Select suits for pair: C(5,2) ways Permute remaining suits on singletons: 3! Ways **Total 13*C(12,3)*C(5,2)*3!=171600 ways** **Probability: 171600/C(65,5)**	.0208	**15**
1 Pair	Start with general pair and subtract Rainbow Pair Select rank for pair: 13 ways Select 3 ranks for singletons: C(12,3) ways Select suits for pair: C(5,2) ways Select suits for 3 singletons: 5^3 ways **subtotal: 13*C(12,3)*C(5,2)*5^3 ways** Subtract Rainbow Pair: -171600 **Total 13*C(12,3)*C(5,2)*(5^3-3!)= 3403400 ways** **Probability: 3403400/C(65,5)**	.412	**18**
Rainbow	Start with general rainbow and subtract all special rainbows Select rank for Hearts: 13 ways Select rank for Spades: 13 ways Select rank for Clubs: 13 ways Select rank for Diamonds: 13 ways Select rank for Bells: 13 ways **subtotal: 13^5 ways** Subtract Rainbow Pair: -171600 Subtract Rainbow 2 Pair: -25740 Subtract Rainbow 3 of a Kind: -2600 Subtract Rainbow Full House: -1560 Rainbow 4 of a Kind: -780 Straight Rainbow: -1080 Subtract Royal Rainbow: -120 Subtract 5 of a Kind: -13 **Total 167800 ways Probability: 167800/C(65,5)**	.0203	**14**