Application of Confidence Intervals:

Value at Risk

Neil Gupta

Overview

How much can I lose if I invest in this security? It is this question that many investors have before jumping into today’s volatile markets. Value at Risk tries to provide an answer, at least within a reasonable bound. Risk management is at the epicenter of discussions in today’s financial markets. There are many risk management models, but by far the most widely used is called VaR or Value at Risk. It is built around statistical topics and probability concepts that have been around for decades. Value at Risk was established and disseminated in the early 1990s by a handful of scientists and mathematicians called “quants.” These were the mathematicians who went to work for [JPMorgan](http://topics.nytimes.com/top/news/business/companies/morgan_j_p_chase_and_company/index.html?inline=nyt-org). VaR had great appeal, it simply expressed a single number to upper management as to the risk level associated with a single security.

VaR isn’t one model but rather a group of related models that share a mathematical framework. In its most common form, it measures the boundaries of risk in a portfolio over short durations, assuming a normally distributed market. VaR calculates the worst expected shortfall over a given time period at a given confidence level. Value at Risk estimates can be calculated for various types of risk: market, credit, and operations. For the purposes of this case study we will only focus on market risk. Therefore, if the Value at Risk on an instrument is 5 million dollars at a one-day, 95% confidence level, there is a only a 5% chance that the value of the asset will drop more than 5 million over any given day. Value at Risk is mostly used by large institutions like investment banks and commercial players. Another reason VaR is useful is that it can calculate both separate risks in a single portfolio and firm wide risk. Top executives usually know their firm’s daily VaR within minutes of the market’s close.

Let’s review a number of Value at Risk’s assumptions. A distribution is normal if there is a high probability that any observation from the population sample will have a value that is close to the mean, and a low probability of having a value that is far from the mean. The normal distribution curve is used by many VaR models, which assume that asset returns follow a normal pattern. Normal distribution tables show the probability of a particular observation moving a certain distance from the mean. If we look along a normal distribution table we see that at -1.645 standard deviations, the probability is 5%; this means that there is a 5% probability that an observation will be at least 1.645 standard deviations below the mean. This level will be used in our VaR model.

There are three methods for calculating Value a Risk.

* Parametric(Correlation Approach)
* Historical Value at Risk
* Monte Carlo Simulations

Using the parametric method, the volatility of each factor is calculated from the price series over the suggested time period. With this time series we then need to calculate a log return. We can calculate correlations between each asset in the portfolio. We would also need to calculate the volatility (Standard Deviation) of the asset. For the purposes of this case study we will be using the last 90 days of data to calculate the volatility. We will be using a front weighted volatility meaning that the most recent observations are more heavily weighted then observations which were older. A major pitfall with this parametric approach is that we assume correlations are constant during our observable period. For calculating a daily VaR that may be sufficient but over long periods it may not be. Our case study will use the parametric approach to Value at Risk. Below is a table of the last 90 day volatilities.



The historical VaR avoids some of the assumptions of the correlation method. For historical simulation the model calculates potential losses using actual historical returns. This means rare events and crashes would be included in the results. As the factor returns used for revaluing the portfolio are actual past movements, the correlations in the calculation are also actual past correlations. This shows that historical VaR may provide a better accuracy for portfolio movements.

The final method, Monte Carlo simulation is the most flexible method of the three. As with historical VaR, Monte Carlo simulation allows the risk manager to use actual historical distributions for risk factor returns rather than having to assume normal returns. A large number of randomly generated simulations are run forward in time using volatility and correlation estimates chosen by the risk manager. Each simulation will be different but in total the simulations will aggregate to the chosen statistical parameters. This method is more realistic than the previous two models and therefore is more likely to estimate VaR more accurately. However its implementation requires powerful computers and there is also a trade-off in that the time required to perform calculations is longer.

**Distributional Assumptions**

In our case study we will analyze a portfolio with Crude Oil, Corn, Soybeans and the S&P 500. Before we jump into replicating a VaR model let’s take a look at the distributions of returns from 1990 in these instruments.

Crude Oil is the most volatile commodity in today’s market. Below you will see the distribution of returns from 1990. There appear to be many outliers in the negative return area which may skew the distribution. The red line represents what a normal distribution would appear to look like. It is clear that crude oil as highly volatile as it is may not be the best example of an asset which moves “normally.”



Next we have the S&P 500. As we can see there is much less range in the values which the distribution can have. Note our distributions are quite wide because this encompasses the bubble leading up to 2008 and the market crash that shortly ensued.



Taking a close look at the soft commodities we have corn. Corn is a very speculative commodity in the US since it is used to many different industries.



Taking a look at soybeans we see a more even distribution.



**Correlation**

Correlation between variables is important to fund managers who need to know in reducing their risk exposure through diversifying their portfolio. Correlation is a measure of the degree to which a value of one variable is related to the value of another. The correlation coefficient is a number that compares the directional strength of the movements in two instruments values. The sign of the coefficient determines the relative directions that the instruments move in, while its value determines the strength of the relative movements. The value of the coefficient ranges from -1 to +1, depending on the nature of the relationship. For example, the value of the correlation is 0.5, this means that one instrument moves in the same direction by half of the amount that the other instrument moves. A value of zero means that the instruments are uncorrelated, and their movements are independent of each other. Below we have created a correlation matrix based upon the last 90 days.



**One Asset Value at Risk**

Our assessment of Value at Risk begins with one asset, Crude Oil. We will hypothetically assume we are long one contract of Crude Oil on the NYMEX exchange. We have already calculated our volatility, correlation, and we are assuming a normal distribution. In the spreadsheet I am attaching with this project I have calculated the VaR. An important method to track your results is backtesting the VaR model. This takes into account the actual change in Crude Oil against our expected change according to our VaR model. This will show us how accurate we really are.



This shows us that the predicted VaR was broken 5.6% of the time since 1990. Notice how the VaR back in the summer of 2008 blew out when Crude Oil went to 140 a barrel and subsequently down to 35.Since VaR is used more towards the downside we can feel comfortable with these results. However we should expect Downside and Upside Breaks to be about 2.5% each if our normal assumption was accurate.

**Multiple Asset Portfolio**

In our second spreadsheet I created a multiple asset portfolio made up of the following positions. With this model we are taking into account the diversification benefit of each of these instruments. By finding the correlation matrix we will be efficiently diversifying away some of the risks we may have. The model shows the difference between calculating value at risk of each position separately and as a whole portfolio.



Conclusion

Value at Risk has developed as a risk assessment tool at banks and other financial service firms in the last decade. Its usage in these firms has been driven by the failure of the risk tracking systems used until the early 90s to detect dangerous risk taking on the part of traders and it offered a key benefit. I have demonstrated two parametric forms of VaR. Many institutions will employ all forms of Value at Risk at many different portfolio levels. This model can be also used for the individual investor in equity markets. At the heart of the model we see that is simply calculating a confidence interval based upon historical movements. I find this model to be fairly easy to understand and also helps to understand how much I can potentially loose when I enter into today’s markets.

Works Cited

Choudhry, M., *The Bond and Money Markets: Strategy, Trading, Analysis*,Butterworth-Heinemann, 2001

Holton, Glyn A. *Value at Risk: Theory and Practice*, Academic Press, 2003

Jorion, Philippe. *Value at Risk,* McGraw Hill, 2000