Math 420 - Pretest

August 21, 2012

1 Biographic Info

Name

Email

Major/Year

Class: 420 520

Are you taking this class for a grade?

What days/times would be best for office hours?

Are there any weeks you cannot attend class?

Research Interests

Familiarity with software, computing

What do you want to get out of this class?

2 Math background

A significant portion of this class involves working through problems as part of a group, either a small group or the entire class. There is also a presentation component to this class where all students will need to learn and then present material to the rest of the class. In an attempt to better organize these group work sessions and presentations, I would like to know about your mathematics background.

Basically, all I want you to do is tell me about the math classes you have taken, and explain the topics you feel comfortable with, and the topics you don't feel comfortable with. If you know alot of algebra, tell me about it, and if you've never taken a class on number theory, tell me about it.

3 Diagnostic questions

This section will help me judge your command of various math topics, and your ability to analyze and construct proofs. Check all the problems before you start, and answer the ones you know first. Try any problems you think you might figure out, and disregard anything that you have no idea about.

Problem 1

Explain each of the steps of this "proof", and explain what is unacceptable about it. Assume $x \neq 0$.

$$x = y$$

$$x^2 = xy$$

$$x^2 - y^2 = xy - y^2$$

$$(x - y)(x + y) = y(x - y)$$

$$x + y = y$$

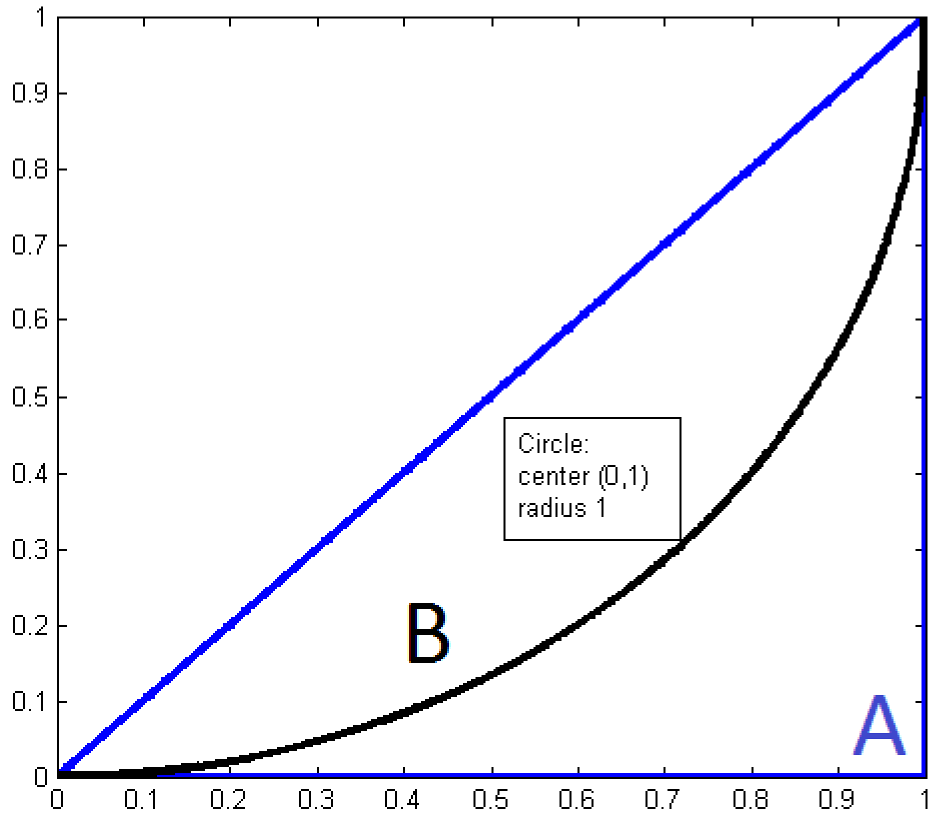
$$y + y = y$$

$$2y = y$$

$$2 = 1$$

Problem 2

Consider the picture below. Determine the sizes of the regions A (bounded by the lines $y = x$, $x = 1$ and $y = 0$) and B (bounded by $y = x$ and the circle centered at $(0, 1)$ with radius 1). Use any mathematics you would like.



Problem 3

A famous "proof" by Lewis Carroll once concluded that all triangles are isoceles triangles (two equal sides). The steps are listed below, and you are asked to follow them with your own triangle and find the fallacy. Obviously, it will be easier to notice if the triangle you draw is not isoceles.

Note: Even if you have trouble with the first question, try the other two questions, since they do not require the solution to 3a.

Problem 3a

Given a triangle $\triangle ABC$, prove that $|AB| = |AC|$. When a side is given in the form $|AB|$, that refers to the length of the side.

1. Draw a line bisecting $\angle A$.
2. Label the midpoint of line segment BC, D.
3. Draw the perpendicular bisector of segment BC, which contains D.
4. If these two lines are parallel, $|AB| = |AC|$ and we are done. Otherwise, they intersect at point O.
5. Draw line OR perpendicular to AB, and line OQ perpendicular to AC.
6. Draw lines OB and OC.
7. By *Angle-Angle-Side* congruency, $\triangle RAO \cong \triangle QAO$.
   - This follows because $\angle OAQ \cong \angle OAR$ (recall OA bisects $\angle A$), $\angle ARO \cong \angle AQO$ (they are both right angles) and $|AO| = |AO|$ of course.
8. By *Side-Angle-Side* congruency, $\triangle ODB \cong \triangle ODC$.
   - This follows because $\angle ODB \cong \angle ODC$ (they are right angles), $|BD| = |CD|$ because the point D bisects BC, and of course $|OD| = |OD|$.
9. By *Side-Side-Angle* congruency, $\triangle ROB \cong \triangle QOC$.
   - This follows because $\angle ORB \cong \angle OQC$ (they are right angles), $|OR| = |OQ|$ (because of the congruency in step 7), and $|OB| = |OC|$ (because of the congruency in step 8).
10. Given all these congruencies $AR \cong AQ$ and $RB \cong QC$. This means that $|AR| + |RB| = |AQ| + |QC|$, or $|AB| = |AC|$.

Problem 3b

Explain what happened in step 4. Why could we conclude if the perpendicular bisector of BC and the angle bisector of $\angle A$ were parallel?

Problem 3c

Explain how you could use a proof that all triangles are isoceles to show that all triangles are equilateral. Assume that every triangle $\triangle ABC$ has sides $|AB| = |AC|$, even though the earlier question had a faulty proof.

Problem 4

A long time ago, Greek people did math. They were good at it, but not as good as we are today. Back then, they considered a race between Achilles (who I guess was quite fast) and a tortoise. The tortoise was given a 10 mile head start, because Achilles can run 10 miles per hour, whereas the tortoise could only run 1 mile per hour.

By the time Achilles ran 10 miles, the tortoise had run a mile further. Once Achilles ran that extra mile, the tortoise (somehow with a perfect sense of direction) had run an extra 1/10 of a mile. After Achilles ran that 1/10 of a mile, the tortoise was still 1/100 of a mile ahead.

The Greeks postulated that, because the tortoise could always stay slightly ahead of Achilles, that Achilles would never catch it, and turn it in to stew or whatever they did with tortoises. Of course, that makes no sense to us, because we understand modern mathematics. Explain what the Greek mathematicians were missing, and tell me after what time and distance traveled Achilles caught the tortoise.