Math 420 - Homework 3

September 11, 2012

These questions do not come from the textbook. I may be posting some relevant material on the class website for your consumption.

1 Stretching

The **parallel postulate** states that: if a straight line falling on two straight lines make the interior angles on the same side less than two right angles, then the two straight lines will at some point intersect on the side on which the angles are less than right angles.

1. Draw a picture of this situation, and confirm that this makes sense.
2. Why is this called the parallel postulate? *Hint: consider the situation where the two angles are right angles.*
3. Draw a trapezoid with two right angles.
   (a) Using the parallel postulate, prove that the remaining two angles must at up to π.
   (b) Suppose the trapezoid had its two parallel sides equal. Can you prove that the other two angles must be right angles without the parallel postulate?

2 Warming Up

1. Describe the following terms, as related to spherical geometry:
   (a) Sphere
   (b) Antipodal point
   (c) Great circle
   (d) Parallel Lines
   (e) Perpendicular Lines
2. Rewrite the following statements in terms of spherical geometry, and include any explanation if necessary:
   (a) A **line segment** is the shortest path between two points.
   (b) There is a unique line passing through **any two distinct points**.
   (c) A straight line **never ends**.
   (d) If three points are collinear, **one point is between the other two**.
   (e) Perpendicular lines form **four** right angles.
3. Describe the three ways that a plane P and a sphere S with center C may intersect.
4. In the event P includes C, the intersection of P and S forms what?
5. What is the length of every great circle on a sphere of radius R?

3 Two man game

Consider two distinct great circles on a sphere. Suppose the angle at which they intersect is $\theta < \pi/2s$. We are interested in the area between these two great circles, which is called a **lune**.

1. What is the relationship between the two angles in a lune.
2. What is needed to prove one lune congruent to another on a sphere? That is to say, what is necessary and sufficient to define a lune. Explain why.
3. What is the area of a lune with angle $\theta < \pi/2s$ on a sphere of radius R.
4. Why can a lune not exist in Euclidean space?

4 Triangle offense

A spherical triangle is defined just like a planar triangle. It consists of three points which are joined by arcs of great circles, and the area that is enclosed within.

1. When you consider the intersection of three distinct lines, how many sections have you now divided the sphere into? *Note:* From now on you can restrict your focus to just the smallest of the eight triangles.
2. In the Euclidean plane, having a line transversal to two lines at a right angle meant that the two lines are parallel. What about on a sphere?
3. Can you create a triangle with all the same angles on a sphere?
4. How do the following tests for congruency apply to triangles on the sphere:
   (a) Angle-Side-Angle
   (b) Side-Angle-Side
   (c) Side-Side-Angle
   (d) Side-Side-Side
5. Are there any new congruency tests we can add for triangles on the sphere?
6. Can there be a triangle with two right angles? What about three?

5 Buzzer beater

1. Find a formula for the area of a spherical triangle? Explain how it works.

6 Overtime

These questions are bonus questions, because they cover material we haven't really talked about. Try them out if you're feeling good.

1. Using the calculus of variations (or even just calculus plus some thinking) prove that the minor arc of a great circle is the shortest distance between two points on a sphere.
2. Two lines cut a sphere into 4 pieces. What is the minimum number of lines would you need to cut the Euclidean plane into 4 pieces? What about 5 pieces?