Math 333 Quiz 2 - February 6, 2013

Question 1

Consider the number $z = -2 + 2\imath$.

Question 1.a

Write this number is polar form. *Hint:* $\tan^{-1}(1) = \pi/4$.

Answer 1.a

We need to write $z = re\imath\theta$, where

$$
\begin{aligned}
r &= \sqrt{(-2)^2 + (2)^2} = \sqrt{8} \\
\theta &= \tan^{-1}(2/(-2)) = -\pi/4
\end{aligned}
$$

Notice that because $\Re(z) < 0$, $\theta = -\pi/4 + \pi = 3\pi/4$. Our answer is

$$z = \sqrt{8}\mathrm{e}^{\imath 3\pi/4}.$$

Question 1.b

Compute all values of $z^{1/3}$.

Answer 1.b

Now that we know the polar form

$$z = \sqrt{8}\mathrm{e}^{\imath(3\pi/4 + 2\pi k)}, \qquad k \in \mathbb{Z},$$

we can simply apply the root to find

$$z^{1/3} = 8^{1/6}\mathrm{e}^{\imath(\pi/4 + 2\pi k/3)}, \qquad k \in \mathbb{Z}.$$

Because we needed the third root, there are 3 distinct solutions, for $k = 0, 1, 2$.

Question 2

These questions all deal with the unit circle.

Question 2.a

How many radians compose one full rotation on the unit circle?

Answer 2.a

There are 2π radians in the unit circle.

Question 2.b

Write an equation, involving the modulus, defining all points z which appear on the unit circle.

Answer 2.b

The points in the unit circle are all distance 1 from the origin, therefore the defining equation is

$$|z| = 1.$$

Question 2.c

Explain why the 3 values of the number $z = (1)^{1/3}$ appear equally spaced around the unit circle.

Answer 2.c

The point 1 can be written in polar form as

$$1 = \mathrm{e}^{\imath 2\pi k}, \qquad k \in \mathbb{Z}.$$

By applying the 1/3 power, $\mathrm{Arg}(1^{1/3}) = 2\pi k/3$, which has three unique values, for $k = -1, 0, 1$. Those three values are evenly spaced around the unit circle, because the integers are equally far apart.