Math 333 Quiz 1 - January 30, 2013

Question 1

This problem deals with the conversion between Cartesian and polar form.

Question 1.a

Write the number $1 - 2\imath$ in exponential (polar) form.

Answer 1.a

We need to find the magnitude and argument of $z = 1 - 2\imath$ so that we can write $z = r\mathrm{e}^{\imath\theta}$.

$$\begin{aligned} r &= |z| = \sqrt{1^2 + (-2)^2} = \sqrt{5}, \\ \theta &= \tan^{-1}(-2/1) = -\tan^{-1} 2. \end{aligned}$$

This means that we can write our complex number as

$$z = \sqrt{5}\exp(-\imath\tan^{-1} 2).$$

If this point were in the left half-plane (with $\Re(z) < 0$) we would need to alter the angle by π. This point is in the right half-plane, which means we are fine.

Question 1.b

Write the number $2\mathrm{e}^{\pi/6\imath}$ in Cartesian form.

Answer 1.b

Using Euler's identity, we can simplify this exponential:

$$\mathrm{e}^{\pi/6\imath} = \cos(\pi/6) + \imath\sin(\pi/6) = \frac{3}{2} + \imath\frac{1}{2}$$

Plugging this in allows us to write $z = 2\mathrm{e}^{\pi/6\imath}$ as

$$\begin{aligned} z &= 2\left(\frac{3}{2} + \imath\frac{1}{2}\right) \\ &= 3 + \imath. \end{aligned}$$

Question 2

Suppose you are given the point $z = 1 + \imath$.

Question 2.a

Compute z^2 and z^3.

Answer 2.a

You can either move this to polar and compute the powers there, or just perform the necessary multiplication. I will do the latter here:

$$\begin{aligned} z^2 &= (1+\imath)(1+\imath) = 2\imath, \\ z^3 &= 2\imath(1+\imath) = -2 + 2\imath. \end{aligned}$$

Question 2.b

Plot z, z^2 and z^3 on the same graph.

Answer 2.b