Residue of a function f at a pole of order n z_0,

$$\text{Res}(f, z_0) = \frac{1}{(n-1)!} \lim_{z \to z_0} \frac{d^{n-1}}{dz^{n-1}} (z - z_0)^n f(z).$$

Residue theorem:

$$\int_C f(z)dz = 2\pi\imath \left(\sum_{k=1}^{n} \text{Res}(f, z_k) \right),$$

where z_k are singularities of f which lie *inside* the contour C.

Inner product for $\boldsymbol{u}, \boldsymbol{v} \in \mathbb{R}^n$

$$\boldsymbol{u}^T \boldsymbol{v} = \sum_{k=1}^{n} u_k v_k = \|u\| \|v\| \cos\theta.$$

Note that the inner product of two vectors is a scalar, i.e., $\boldsymbol{u}^T \boldsymbol{v} \in \mathbb{R}$. This is different than outer product $\boldsymbol{u}\boldsymbol{v}^T$, which is a matrix of size $n \times n$.

Multiplying two outer products is equal to

$$\begin{aligned}(\boldsymbol{u}\boldsymbol{v}^T)(\boldsymbol{u}\boldsymbol{v}^T) &= \boldsymbol{u}(\boldsymbol{v}^T\boldsymbol{u})\boldsymbol{v}^T \\ &= (\boldsymbol{v}^T\boldsymbol{u})(\boldsymbol{u}\boldsymbol{v}^T)\end{aligned}$$

because $\boldsymbol{v}^T\boldsymbol{u}$ is a *scalar.*

Norm of a vector:

$$\|\boldsymbol{u}\| = \sqrt{\boldsymbol{u}^T\boldsymbol{u}}.$$

Two vectors are *orthogonal* if $\boldsymbol{u}^T\boldsymbol{v} = 0$.

$\boldsymbol{u}$ is a *unit vector* if $\|\boldsymbol{u}\| = 1$.

A set of vectors $B = \{\boldsymbol{u}_1, \ldots, \boldsymbol{u}_k\}$ are *linearly independent* if and only if

$$c_1\boldsymbol{u}_1 + \ldots + c_k\boldsymbol{u}_k = 0$$

only when $c_1 = \ldots = c_k = 0$.

The $n \times n$ *identity matrix* I_n has all zeros, except for ones on the diagonal. For a matrix $\mathsf{A} \in \mathbb{R}^{n \times n}$ and vector $\boldsymbol{x} \in \mathbb{R}^n$, the following are true:

$$\mathsf{I}_n\mathsf{A} = \mathsf{A}, \qquad \mathsf{I}_n\boldsymbol{x} = \boldsymbol{x}.$$

A matrix A is *symmetric* if $\mathsf{A} = \mathsf{A}^T$.

The *LU decomposition* $\mathsf{A} = \mathsf{LU}$ finds a lower triangular matrix L and an upper triangular matrix U. The diagonal values on L are all 1.

If you know $\mathsf{A} = \mathsf{LU}$, you can easily solve a system $\mathsf{A}\boldsymbol{x} = \boldsymbol{b}$ by solving two triangular systems:

$$\begin{aligned}\mathsf{L}\boldsymbol{y} &= \boldsymbol{b}, \\ \mathsf{U}\boldsymbol{x} &= \boldsymbol{y}.\end{aligned}$$

The *inverse* of a matrix $\mathsf{A} \in \mathbb{R}^{n \times n}$ is defined as

$$\mathsf{A}\mathsf{A}^{-1} = \mathsf{A}^{-1}\mathsf{A} = \mathsf{I}_n$$

where I_n is the $n \times n$ identity matrix.

Basic matrix properties

$$\begin{aligned}(\mathsf{AB})^T &= \mathsf{B}^T\mathsf{A}^T \\ (\mathsf{A}+\mathsf{B})^T &= \mathsf{A}^T + \mathsf{B}^T \\ (\mathsf{AB})^{-1} &= \mathsf{B}^{-1}\mathsf{A}^{-1} \\ (\mathsf{A}+\mathsf{B})(\mathsf{C}+\mathsf{D}) &= \mathsf{AC} + \mathsf{AD} + \mathsf{BC} + \mathsf{BD}\end{aligned}$$

A matrix V is *orthogonal* if $\mathsf{V}^T\mathsf{V} = \mathsf{I}_n$.

The eigenvalue equation is

$$\mathsf{A}\boldsymbol{x} = \lambda\boldsymbol{x}.$$

$\boldsymbol{x}$ is an *eigenvector* and λ is an *eigenvalue.*

If A is symmetric, the eigenvectors of A are orthogonal.

The spectral decomposition of a matrix A is

$$\mathsf{A} = \mathsf{X}\Lambda\mathsf{X}^{-1}.$$