Assume below that $z = x + \imath y$.

Basics

Size of a complex number

$$|z| = \sqrt{x^2 + y^2}$$

Circle centered at z_0 with radius ρ

$$|z - z_0| = \rho$$

Complex function

$$f(z) = u(x, y) + \imath v(x, y)$$

Euler's formula

$$\mathrm{e}^z = \mathrm{e}^x(\cos y + \imath \sin y)$$

Principal argument

$$\tan \operatorname{Arg} z = \frac{y}{x}, \quad \operatorname{Arg} z \in (-\pi, \pi]$$

Complex logarithm

$$\ln z = \log|z| + \imath(\operatorname{Arg} z + 2\pi k), \quad k \in \mathbb{Z}$$

Inverse trig (recall $z^{1/2}$ has 2 values)

$$\sin^{-1} z = -\imath \ln[\imath z + (1 - z^2)^{1/2}]$$

Cauchy-Riemann Equations

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \qquad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

Integrals

Parametrized contour integral (C as $z(t)$)

$$\int_C f(z)dz = \int_a^b f(z(t))z'(t)dt, \quad a \leq t \leq b$$

Basic Integral Theorem

$$\oint \frac{1}{(z - z_0)^n} = \begin{cases} 2\pi\imath & n = 1 \\ 0 & n \neq 1 \end{cases}$$

Independence of path theorem

$$\int_C f(z)dz = F(z_1) - F(z_0), \quad \frac{dF}{dz} = f$$

Cauchy's Integral Formula

$$\oint \frac{f(z)}{(z - z_0)^{n+1}} dz = \frac{2\pi\imath}{n!} f^{(n)}(z_0)$$

Series

Taylor series

$$f(z) = \sum_{k=0}^{\infty} \frac{f^{(k)}(z_0)}{k!}(z - z_0)^k$$

Power series

$$\frac{1}{1 - z} = \sum_{k=0}^{\infty} z^k, \quad |z| < 1$$

Exponential series

$$\mathrm{e}^z = \sum_{k=0}^{\infty} \frac{z^k}{k!}$$

Cosine series

$$\cos z = \sum_{k=0}^{\infty} (-1)^k \frac{z^{2k}}{(2k)!}$$

Sine series

$$\sin z = \sum_{k=0}^{\infty} (-1)^k \frac{z^{2k+1}}{(2k+1)!}$$