

Math 333 HW #1

Due: Sept. 5

Chapter 17.1

Write the given number in $x + iy$ form.

2. $3i^5 - i^4 + 7i^3 - 10i^2 - 9$

3. i^8

6. $3(4 - i) - 3(5 + 2i)$

7. $i(5 + 7i)$

10. $(\frac{1}{2} - \frac{1}{4}i)(\frac{2}{3} + \frac{5}{3}i)$

11. $(2 + 3i)^2$

13.
$$\frac{2}{i}$$

16.
$$\frac{10 - 5i}{6 + 2i}$$

19.
$$\frac{(5 - 4i) - (3 + 7i)}{(4 + 2i) + (2 - 3i)}$$

Find the indicated expression, where $z = x + iy$. Note that $\Re(z)$ means the real part of z and $\Im(z)$ means the imaginary part of z .

27. $\Re(1/z)$

28. $\Re(z^2)$

30. $\Im(\bar{z}^2 + z^2)$

31. $|z - 1 - 3i|$

Solve the following equations, assuming $z = x + iy$, and recalling that $z_1 = z_2$ if and only if $x_1 = x_2$ and $y_1 = y_2$.

33. $2z = i(2 + 9i)$

36. $\bar{z}^2 = 4z$

Chapter 17.2

Write the given number in polar form.

1. 2

4. $6i$

6. $5 - 5i$

7. $-\sqrt{3} + i$

9. $3/(-1 + i)$

Write the number in $x + iy$ form.

11. $z = 5 \exp(i7\pi/6)$

14. $z = 10 \exp(i\pi/5)$

Find the following products or quotients using polar form.

15. $z_1/z_2, z_1 = 2e^{i\pi/8}, z_2 = 4e^{i3\pi/8}$

17. $z_1 z_2, z_1 = 3 - 3i, z_2 = 5 + i5\sqrt{3}$

20.
$$\frac{\sqrt{2} + i\sqrt{6}}{-1 + i\sqrt{3}}$$

Compute the indicated power.

21. $(1 + i\sqrt{3})^9$

24. $(-\sqrt{2} + i\sqrt{6})^4$

25.
$$\left(\exp\left(i\frac{\pi}{8}\right)\right)^{12}$$

26.
$$\left(\sqrt{3} \exp\left(i\frac{2\pi}{9}\right)\right)^6$$

Find all the roots indicated.

27. $8^{1/3}$
28. $1^{1/8}$
31. $(-1 + i\sqrt{3})^{1/2}$

Determine the validity of the following statement:

39a. If $z_1 = -1$ and $z_2 = 5i$,

$$\text{Arg}(z_1 z_2) = \text{Arg}(z_1) + \text{Arg}(z_2).$$

Chapter 17.3

Sketch the graph of the given equation.

1. $\Re(z) = 5$
3. $\Im(\bar{z} + 3i) = 6$
4. $\Im(z - i) = \Re(z + 4 - 3i)$
5. $|z - 3i| = 2$

Sketch the set of points in the complex plane satisfying the given inequality.

9. $\Re(z) < -1$
12. $\Im(z - i) < 5$
13. $2 < \Re(z - 1) < 4$
15. $\Re(z^2) > 0$
16. $\Im(1/z) < 1/2$
19. $|z - i| > 1$
22. $1 \leq |z - 1 - i| < 2$
25. Describe the set of points in the complex plane that satisfies $z^2 + \bar{z}^2 = 2$.

Chapter 17.4

Find the image of the given line under the mapping $f(z) = z^2$.

1. $y = 2$

3. $x = 0$
6. $y = -x$

Express the given function in the form $f(z) = u + iv$.

7. $f(z) = 6z - 5 + 9i$
8. $f(z) = 7z - 9i\bar{z} - 3 + 2i$
10. $f(z) = 3\bar{z}^2 + 2z$
13. $f(z) = z + 1/z$

Evaluate the given function at the indicated points.

15. $f(z) = 2x - y^2 + i(xy^3 - 2x^2 + 1)$
 - (a) $2i$
 - (b) $2 - i$
 - (c) $5 + 3i$

Find the limit.

19. $\lim_{z \rightarrow i} 4z^3 - 5z^2 + 4z + 1 - 5i$
20. $\lim_{z \rightarrow 1-i} \frac{5z^2 - 2z + 2}{z + 1}$

Use differentiation rules to find $f'(z)$.

27. $f(z) = 4z^3 - (3 + i)z^2 - 5z + 4$
33. $f(z) = \frac{3z - 4 + 8i}{2z + i}$

Give the points at which the given function will not be analytic.

35. $f(z) = \frac{z}{z - 3i}$
36. $f(z) = \frac{2i}{z^2 - 2z + 5iz}$