Math 121

Quiz #6

Problem 1

Find the following antiderivative

$$f(x) = \int \left(7 + e^{-2x} + 3x^2 + \frac{1}{x^2}\right) dx$$

Answer

As we did for derivatives, we need to break up integrals into pieces. In this case, each of the pieces can be handled with the power rule

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + C, \qquad n \neq -1$$

except the exponential term which is handled with the rule

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + C$$

Using these two rules where appropriate, and recalling that $\frac{1}{x^2}$ can be written as x^{-2}, we see that

$$f(x) = 7x - \frac{1}{2} e^{-2x} + x^3 - \frac{1}{x} + C$$

Problem 2

Find $f(x)$ given the following information:

$$f(x) = \int \left(-x^2 + 2x\right) dx$$

and

$$f(1) = 1.$$

Answer

This problem comes in two parts: first you have to perform the integral and then you need to use the given condition to determine the unique C associated with f. The integral itself is trivial, requiring only the power rule (refer to problem 1):

$$f(x) = -\frac{1}{3}x^3 + x^2 + C$$

Now we have f along with a C term present. Let's get rid of that C term with the condition

$$\begin{aligned} f(1) &= 1 \\ f(1) &= -\frac{1}{3}(1)^3 + (1)^2 + C \\ 1 &= -\frac{1}{3}(1)^3 + (1)^2 + C \\ 1 &= -\frac{1}{3} + 1 + C \\ \frac{1}{3} &= C \end{aligned}$$

Using this unique value of C we can write

$$f(x) = -\frac{1}{3}x^3 + x^2 + \frac{1}{3}$$