

Math 121

Quiz #4

Problem 1

Problem 1.a

What is the definition of continuity? (*Hint: Two things should be set equal*)

Answer

A function f is continuous at a point a if $f(a)$ exists and

$$f(a) = \lim_{x \rightarrow a} f(x)$$

Problem 1.b

Consider the function

$$f(x) = \log x + \sqrt{\left(\frac{1}{1+x^2}\right)}.$$

Is it continuous at $x = 2$?

Answer

A function is discontinuous in this class under 4 circumstances:

- Division by 0
- Square root of -1
- Logarithm of 0
- Case-Defined break

These are the circumstances when either a function value doesn't exist or the limit doesn't exist. When we look at $x = 2$, none of those issues arise, therefore f is continuous at $x = 2$.

Problem 2

Solve the inequality

$$\frac{x^2 - 4}{x} < 0$$

Answer

We need to find locations where $f(x) = (x^2 - 4)/x$ is either equal to 0 or discontinuous. We solve

$$x^2 - 4 = 0 \quad \Rightarrow \quad x = \{-2, 2\}$$

and determine that the only discontinuity for this function occurs when $x = 0$ and we attempt to divide by 0. Given that, we divide the number line into 4 sections and test points in each of those sections to determine if the function f is positive or negative: The point of this problem was to

	$(-\infty, -2)$	$(-2, 0)$	$(0, 2)$	$(2, \infty)$
Test x	-3	-1	1	3
$f(x)$	$-5/3$	3	-3	$5/3$
$f(x) < 0$	✓		✓	

determine where $f(x) < 0$, and the answer is

$$x \in (-\infty, -2) \cup (0, 2)$$