EXAMPLE 7 An Integral Involving $\frac{1}{u}\,du$

Find $\displaystyle\int \frac{(2x^3+3x)\,dx}{x^4+3x^2+7}$.

Solution: If $u = x^4 + 3x^2 + 7$, then $du = (4x^3 + 6x)\,dx$, which is two times the numerator giving $(2x^3 + 3x)\,dx = \dfrac{du}{2}$. To apply Equation (3), we write

$$\begin{aligned}\int \frac{2x^3+3x}{x^4+3x^2+7}\,dx &= \frac{1}{2}\int \frac{1}{u}\,du \\ &= \frac{1}{2}\ln|u| + C \\ &= \frac{1}{2}\ln|x^4+3x^2+7| + C \qquad \text{(Rewrite } u \text{ in terms of } x.\text{)} \\ &= \frac{1}{2}\ln(x^4+3x^2+7) + C \qquad (x^4+3x^2+7 > 0 \quad \text{for all } x)\end{aligned}$$

NOW WORK PROBLEM 51

EXAMPLE 8 An Integral Involving Two Forms

Find $\displaystyle\int \left(\frac{1}{(1-w)^2} + \frac{1}{w-1}\right) dw$.

Solution:

$$\begin{aligned}\int \left(\frac{1}{(1-w)^2} + \frac{1}{w-1}\right) dw &= \int (1-w)^{-2}dw + \int \frac{1}{w-1}dw \\ &= -1\int (1-w)^{-2}[-dw] + \int \frac{1}{w-1}dw\end{aligned}$$

The first integral has the form $\int u^{-2}\,du$, and the second has the form $\displaystyle\int \frac{1}{v}\,dv$. Thus,

$$\begin{aligned}\int \left(\frac{1}{(1-w)^2} + \frac{1}{w-1}\right) dw &= -\frac{(1-w)^{-1}}{-1} + \ln|w-1| + C \\ &= \frac{1}{1-w} + \ln|w-1| + C\end{aligned}$$

For your convenience, we list in Table 14.2 the basic integration formulas so far discussed. We assume that u is a function of x.

TABLE 14.2 Basic Integration Formulas

1. $\displaystyle\int k\,du = ku + C$	k a constant
2. $\displaystyle\int u^n\,du = \frac{u^{n+1}}{n+1} + C$	$n \neq -1$
3. $\displaystyle\int \frac{1}{u}\,du = \ln\lvert u\rvert + C$	$u \neq 0$
4. $\displaystyle\int e^u\,du = e^u + C$	
5. $\displaystyle\int kf(x)\,dx = k\int f(x)\,dx$	k a constant
6. $\displaystyle\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$	

Problems 14.4

In Problems 1–80, find the indefinite integrals.

1. $\displaystyle\int (x+5)^7\,dx$
2. $\displaystyle\int 15(x+2)^4\,dx$
*3. $\displaystyle\int 2x(x^2+3)^5\,dx$
4. $\displaystyle\int (3x^2+10x)(x^3+5x^2+6)\,dx$
*5. $\displaystyle\int (3y^2+6y)(y^3+3y^2+1)^{2/3}\,dy$
6. $\displaystyle\int (15t^2-6t+1)(5t^3-3t^2+t)^{17}\,dt$
7. $\displaystyle\int \frac{5}{(3x-1)^3}\,dx$
8. $\displaystyle\int \frac{4x}{(2x^2-7)^{10}}\,dx$
9. $\displaystyle\int \sqrt{2x-1}\,dx$
10. $\displaystyle\int \frac{1}{\sqrt{x-5}}\,dx$
11. $\displaystyle\int (7x-6)^4\,dx$
12. $\displaystyle\int x^2(3x^3+7)^3\,dx$
13. $\displaystyle\int u(5u^2-9)^{14}\,du$
14. $\displaystyle\int 9x\sqrt{1+2x^2}\,dx$
*15. $\displaystyle\int 4x^4(27+x^5)^{1/3}\,dx$
16. $\displaystyle\int (4-5x)^9\,dx$
17. $\displaystyle\int 3e^{3x}\,dx$
18. $\displaystyle\int 5e^{3t+7}\,dt$
19. $\displaystyle\int (2t+1)e^{t^2+t}\,dt$
20. $\displaystyle\int -3w^2e^{-w^3}\,dw$
21. $\displaystyle\int xe^{7x^2}\,dx$
22. $\displaystyle\int x^3e^{4x^4}\,dx$
23. $\displaystyle\int 4e^{-3x}\,dx$
24. $\displaystyle\int x^4e^{-6x^5}\,dx$
25. $\displaystyle\int \frac{1}{x+5}\,dx$
26. $\displaystyle\int \frac{12x^2+4x+2}{x+x^2+2x^3}\,dx$
27. $\displaystyle\int \frac{3x^2+4x^3}{x^3+x^4}\,dx$
28. $\displaystyle\int \frac{6x^2-6x}{1-3x^2+2x^3}\,dx$
29. $\displaystyle\int \frac{6z}{(z^2-6)^5}\,dx$
30. $\displaystyle\int \frac{3}{(5v-1)^4}\,dv$
*31. $\displaystyle\int \frac{4}{x}\,dx$
32. $\displaystyle\int \frac{3}{1+2y}\,dy$
33. $\displaystyle\int \frac{s^2}{s^3+5}\,ds$
34. $\displaystyle\int \frac{2x^2}{3-4x^3}\,dx$
35. $\displaystyle\int \frac{5}{4-2x}\,dx$
36. $\displaystyle\int \frac{7t}{5t^2-6}\,dt$
37. $\displaystyle\int \sqrt{5x}\,dx$
38. $\displaystyle\int \frac{1}{(3x)^6}\,dx$
39. $\displaystyle\int \frac{x}{\sqrt{x^2-4}}\,dx$
40. $\displaystyle\int \frac{9}{1-3x}\,dx$
*41. $\displaystyle\int 2y^3e^{y^4+1}\,dx$
42. $\displaystyle\int 2\sqrt{2x-1}\,dx$
43. $\displaystyle\int v^2e^{-2v^3+1}\,dv$
44. $\displaystyle\int \frac{x^2}{\sqrt[3]{2x^3+9}}\,dx$
45. $\displaystyle\int (e^{-5x}+2e^x)\,dx$
46. $\displaystyle\int 4\sqrt[3]{y+1}\,dy$
47. $\displaystyle\int (8x+10)(7-2x^2-5x)^3\,dx$
48. $\displaystyle\int 2ye^{3y^2}\,dy$
49. $\displaystyle\int \frac{x^2+2}{x^3+6x}\,dx$
50. $\displaystyle\int (e^x+2e^{-3x}-e^{5x})\,dx$
*51. $\displaystyle\int \frac{16s-4}{3-2s+4s^2}\,ds$
52. $\displaystyle\int (6t^2+4t)(t^3+t^2+1)^6\,dt$
53. $\displaystyle\int x(2x^2+1)^{-1}\,dx$
54. $\displaystyle\int (8w^5+w^2-2)(6w-w^3-4w^6)^{-4}\,dw$
55. $\displaystyle\int -(x^2-2x^5)(x^3-x^6)^{-10}\,dx$
56. $\displaystyle\int \frac{3}{5}(v-2)e^{2-4v+v^2}\,dv$
57. $\displaystyle\int (2x^3+x)(x^4+x^2)\,dx$
58. $\displaystyle\int (e^{3.1})^2\,dx$
59. $\displaystyle\int \frac{7+14x}{(4-x-x^2)^5}\,dx$
60. $\displaystyle\int (e^x-e^{-x})^2\,dx$
61. $\displaystyle\int x(2x+1)e^{4x^3+3x^2-4}\,dx$
62. $\displaystyle\int (u^3-ue^{6-3u^2})\,du$
63. $\displaystyle\int x\sqrt{(8-5x^2)^3}\,dx$
64. $\displaystyle\int e^{-x/7}\,dx$
65. $\displaystyle\int \left(\sqrt{2x}-\frac{1}{\sqrt{2x}}\right) dx$
66. $\displaystyle\int 3\frac{x^4}{e^{x^5}}\,dx$
*67. $\displaystyle\int (x^2+1)^2\,dx$
68. $\displaystyle\int \left[x(x^2-16)^2 - \frac{1}{2x+5}\right] dx$
69. $\displaystyle\int \left[\frac{x}{x^2+1} + \frac{x^5}{(x^6+1)^2}\right] dx$
70. $\displaystyle\int \left[\frac{3}{x-1} + \frac{1}{(x-1)^2}\right] dx$
71. $\displaystyle\int \left[\frac{2}{4x+1} - (4x^2-8x^5)(x^3-x^6)^{-8}\right] dx$
72. $\displaystyle\int (r^3+5)^2\,dr$
73. $\displaystyle\int \left[\sqrt{3x+1} - \frac{x}{x^2+3}\right] dx$
74. $\displaystyle\int \left[\frac{x}{3x^2+5} - \frac{x^2}{(x^3+1)^3}\right] dx$
75. $\displaystyle\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$
76. $\displaystyle\int (e^5-3^e)\,dx$
77. $\displaystyle\int \frac{1+e^{2x}}{4e^x}\,dx$
78. $\displaystyle\int \frac{2}{t^2}\sqrt{\frac{1}{t}+9}\,dt$
79. $\displaystyle\int \frac{x+1}{x^2+2x}\ln(x^2+2x)\,dx$
80. $\displaystyle\int \sqrt[3]{x}e^{\sqrt[3]{8x^4}}\,dx$

In Problems 81–84, find y subject to the given conditions.

81. $y' = (3-2x)^2$; $y(0) = 1$
82. $y' = \dfrac{x}{x^2+6}$; $y(1) = 0$