Since dr/dq is the derivative of total revenue r,

$$r = \int (2000 - 20q - 3q^2)\,dq$$

$$= 2000q - (20)\frac{q^2}{2} - (3)\frac{q^3}{3} + C$$

so that

$$r = 2000q - 10q^2 - q^3 + C \tag{7}$$

Revenue is 0 when q is 0.

We assume that **when no units are sold, there is no revenue;** that is, $r = 0$ when $q = 0$. This is our initial condition. Putting these values into Equation (7) gives

$$0 = 2000(0) - 10(0)^2 - 0^3 + C$$

Although $q = 0$ gives $C = 0$, this is not true in general. It occurs in this section because the revenue functions are polynomials. In later sections, evaluating at $q = 0$ may produce a nonzero value for C.

Hence, $C = 0$, and

$$r = 2000q - 10q^2 - q^3$$

To find the demand function, we use the fact that $p = r/q$ and substitute for r:

$$p = \frac{r}{q} = \frac{2000q - 10q^2 - q^3}{q}$$

$$p = 2000 - 10q - q^2$$

NOW WORK PROBLEM 11

EXAMPLE 5 Finding Cost from Marginal Cost

In the manufacture of a product, fixed costs per week are \$4000. (Fixed costs are costs, such as rent and insurance, that remain constant at all levels of production during a given time period.) If the marginal-cost function is

$$\frac{dc}{dq} = 0.000001(0.002q^2 - 25q) + 0.2$$

where c is the total cost (in dollars) of producing q pounds of product per week, find the cost of producing 10,000 *lb in* 1 *week.*

Solution: Since dc/dq is the derivative of the total cost c,

$$c(q) = \int [0.000001(0.002q^2 - 25q) + 0.2]\,dq$$

$$= 0.000001 \int (0.002q^2 - 25q)\,dq + \int 0.2\,dq$$

$$c(q) = 0.000001\left(\frac{0.002q^3}{3} - \frac{25q^2}{2}\right) + 0.2q + C$$

When q is 0, total cost is equal to fixed cost.

Fixed costs are constant regardless of output. Therefore, when $q = 0$, $c = 4000$, which is our initial condition. Putting $c(0) = 4000$ in the last equation, we find that $C = 4000$, so

$$c(q) = 0.000001\left(\frac{0.002q^3}{3} - \frac{25q^2}{2}\right) + 0.2q + 4000 \tag{8}$$

Although $q = 0$ gives C a value equal to fixed costs, this is not true in general. It occurs in this section because the cost functions are polynomials. In later sections, evaluating at $q = 0$ may produce a value for C that is different from fixed cost.

From Equation (8), we have $c(10{,}000) = 5416\frac{2}{3}$. Thus, the total cost for producing 10,000 pounds of product in 1 week is \$5416.67.

NOW WORK PROBLEM 15

Problems 14.3

In Problems 1 and 2, find y subject to the given conditions.

*1. $dy/dx = 3x - 4$; $y(-1) = \frac{13}{2}$

2. $dy/dx = x^2 - x$; $y(3) = \frac{19}{2}$

In Problems 3 and 4, if y satisfies the given conditions, find y(x) for the given value of x.

3. $y' = 5/\sqrt{x}$, $y(9) = 50$; $x = 16$

4. $y' = -x^2 + 2x$, $y(2) = 1$; $x = 1$

In Problems 5–8, find y subject to the given conditions.

*5. $y'' = -3x^2 + 4x$; $y'(1) = 2$, $y(1) = 3$

6. $y'' = x + 1$; $y'(0) = 0$, $y(0) = 5$

7. $y''' = 2x$; $y''(-1) = 3$, $y'(3) = 10$, $y(0) = 13$

8. $y''' = e^x + 1$; $y''(0) = 1$, $y'(0) = 2$, $y(0) = 3$

In Problems 9–12, dr/dq is a marginal-revenue function. Find the demand function.

9. $dr/dq = 0.7$

10. $dr/dq = 10 - \frac{1}{16}q$

*11. $dr/dq = 275 - q - 0.3q^2$

12. $dr/dq = 5{,}000 - 3(2q + 2q^3)$

In Problems 13–16, dc/dq is a marginal-cost function and fixed costs are indicated in braces. For Problems 13 and 14, find the total-cost function. For Problems 15 and 16, find the total cost for the indicated value of q.

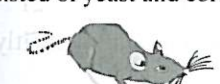
13. $dc/dq = 1.35$; {200}

14. $dc/dq = 2q + 75$; {2000}

*15. $dc/dq = 0.08q^2 - 1.6q + 6.5$; {8000}; $q = 25$

16. $dc/dq = 0.000204q^2 - 0.046q + 6$; {15,000}; $q = 200$

*17. **Diet for Rats** A group of biologists studied the nutritional effects on rats that were fed a diet containing 10% protein.[2] The protein consisted of yeast and corn flour.



Over a period of time, the group found that the (approximate) rate of change of the average weight gain G (in grams) of a rat with respect to the percentage P of yeast in the protein mix was

$$\frac{dG}{dP} = -\frac{P}{25} + 2 \qquad 0 \le P \le 100$$

If $G = 38$ when $P = 10$, find G.

18. **Winter Moth** A study of the winter moth was made in Nova Scotia.[3] The prepupae of the moth fall onto the ground from host trees. It was found that the (approximate) rate at which prepupal density y (the number of prepupae per square foot of soil) changes with respect to distance x (in feet) from the base of a host tree is

$$\frac{dy}{dx} = -1.5 - x \qquad 1 \le x \le 9$$

If $y = 57.3$ when $x = 1$, find y.

19. **Fluid Flow** In the study of the flow of fluid in a tube of constant radius R, such as blood flow in portions of the body, one can think of the tube as consisting of concentric tubes of radius r, where $0 \le r \le R$. The velocity v of the fluid is a function of r and is given by[4]

$$v = \int -\frac{(P_1 - P_2)r}{2l\eta}\,dr$$

where P_1 and P_2 are pressures at the ends of the tube, η (a Greek letter read "eta") is fluid viscosity, and l is the length of the tube. If $v = 0$ when $r = R$, show that

$$v = \frac{(P_1 - P_2)(R^2 - r^2)}{4l\eta}$$

20. **Elasticity of Demand** The sole producer of a product has determined that the marginal-revenue function is

$$\frac{dr}{dq} = 100 - 3q^2$$

Determine the point elasticity of demand for the product when $q = 5$. (*Hint:* First find the demand function.)

21. **Average Cost** A manufacturer has determined that the marginal-cost function is

$$\frac{dc}{dq} = 0.003q^2 - 0.4q + 40$$

where q is the number of units produced. If marginal cost is \$27.50 when $q = 50$ and fixed costs are \$5000, what is the *average* cost of producing 100 units?

22. If $f''(x) = 30x^4 + 12x$ and $f'(1) = 10$, evaluate

$$f(965.335245) - f(-965.335245)$$

OBJECTIVE

To learn and apply the formulas for $\int u^n\,du$, $\int e^u\,du$, and $\int \frac{1}{u}\,du$.

14.4 More Integration Formulas

Power Rule for Integration

The formula

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \qquad \text{if } n \ne -1$$

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[2] Adapted from R. Bressani, "The Use of Yeast in Human Foods," in *Single-Cell Protein*, ed. R. I. Mateles and S. R. Tannenbaum (Cambridge, MA: MIT Press, 1968).

[3] Adapted from D. G. Embree, "The Population Dynamics of the Winter Moth in Nova Scotia, 1954–1962," *Memoirs of the Entomological Society of Canada*, no. 46 (1965).

[4] R. W. Stacy et al., *Essentials of Biological and Medical Physics* (New York: McGraw-Hill, 1955).