Sometimes, in order to apply the basic integration formulas, it is necessary first to perform algebraic manipulations on the integrand, as Example 8 shows.

EXAMPLE 8 Using Algebraic Manipulation to Find an Indefinite Integral

Find $\int y^2\left(y+\frac{2}{3}\right)dy$.

Solution: The integrand does not fit a familiar integration form. However, by multiplying the integrand we get

$$\int y^2\left(y+\frac{2}{3}\right)dy = \int\left(y^3+\frac{2}{3}y^2\right)dy$$

$$= \frac{y^4}{4}+\left(\frac{2}{3}\right)\frac{y^3}{3}+C = \frac{y^4}{4}+\frac{2y^3}{9}+C$$

NOW WORK PROBLEM 41

CAUTION

In Example 8, we first multiplied the factors in the integrand. The answer could not have been found simply in terms of $\int y^2\,dy$ and $\int(y+\frac{2}{3})\,dy$. There is not a formula for the integral of a general product of functions.

EXAMPLE 9 Using Algebraic Manipulation to Find an Indefinite Integral

a. *Find* $\int\frac{(2x-1)(x+3)}{6}\,dx$.

Solution: By factoring out the constant $\frac{1}{6}$ and multiplying the binomials, we get

$$\int\frac{(2x-1)(x+3)}{6}\,dx = \frac{1}{6}\int(2x^2+5x-3)\,dx$$

$$= \frac{1}{6}\left((2)\frac{x^3}{3}+(5)\frac{x^2}{2}-3x\right)+C$$

$$= \frac{x^3}{9}+\frac{5x^2}{12}-\frac{x}{2}+C$$

b. *Find* $\int\frac{x^3-1}{x^2}\,dx$.

Solution: We can break up the integrand into fractions by dividing each term in the numerator by the denominator:

$$\int\frac{x^3-1}{x^2}\,dx = \int\left(\frac{x^3}{x^2}-\frac{1}{x^2}\right)dx = \int(x-x^{-2})\,dx$$

$$= \frac{x^2}{2}-\frac{x^{-1}}{-1}+C = \frac{x^2}{2}+\frac{1}{x}+C$$

NOW WORK PROBLEM 49

Another algebraic approach to part (b) is

$$\int\frac{x^3-1}{x^2}\,dx = \int(x^3-1)x^{-2}\,dx$$

$$= \int(x-x^{-2})\,dx$$

and so on.

Problems 14.2

In Problems 1–52, find the indefinite integrals.

*1. $\int 7\,dx$

2. $\int\frac{1}{2x}\,dx$

*3. $\int x^8\,dx$

4. $\int 5x^{24}\,dx$

*5. $\int 5x^{-7}\,dx$

6. $\int\frac{z^{-3}}{3}\,dz$

7. $\int\frac{2}{x^{10}}\,dx$

8. $\int\frac{7}{x^4\,dx}$

*9. $\int\frac{1}{t^{7/4}}\,dt$

10. $\int\frac{7}{2x^{9/4}}\,dx$

*11. $\int(4+t)\,dt$

12. $\int(r^3+2r)\,dr$

13. $\int(y^5-5y)\,dy$

14. $\int(5-2w-6w^2)\,dw$

*15. $\int(3t^2-4t+5)\,dt$

16. $\int(1+t^2+t^4+t^6)\,dt$

17. $\int(7+e)\,dx$

18. $\int(5-2^{-1})\,dx$

19. $\int\left(\frac{x}{7}-\frac{3}{4}x^4\right)dx$

20. $\int\left(\frac{2x^2}{7}-\frac{8}{3}x^4\right)dx$

*21. $\int\pi e^x\,dx$

22. $\int\left(\frac{e^x}{3}+2x\right)dx$

23. $\int(x^{8.3}-9x^6+3x^{-4}+x^{-3})\,dx$

24. $\int(0.7y^3+10+2y^{-3})\,dy$

25. $\int\frac{-2\sqrt{x}}{3}\,dx$

26. $\int dz$

27. $\int\frac{1}{4\sqrt[8]{x^2}}\,dx$

28. $\int\frac{-4}{(3x)^3}\,dx$

29. $\int\left(\frac{x^3}{3}-\frac{3}{x^3}\right)dx$

30. $\int\left(\frac{1}{2x^3}-\frac{1}{x^4}\right)dx$

31. $\int\left(\frac{3w^2}{2}-\frac{2}{3w^2}\right)dw$

32. $\int\frac{4}{e^{-s}}\,ds$

33. $\int\frac{3u-4}{5}\,du$

34. $\int\frac{1}{12}\left(\frac{1}{3}e^x\right)dx$

35. $\int(u^e+e^u)\,du$

36. $\int\left(3y^3-2y^2+\frac{e^y}{6}\right)dy$

37. $\int(2\sqrt{x}-3\sqrt[4]{x})\,dx$

38. $\int 0\,dt$

39. $\int\left(-\frac{\sqrt[3]{x^2}}{5}-\frac{7}{2\sqrt{x}}+6x\right)dx$

40. $\int\left(\sqrt[3]{u}+\frac{1}{\sqrt{u}}\right)du$

*41. $\int(x^2+5)(x-3)\,dx$

42. $\int x^4(x^3+8x^2+7)\,dx$

43. $\int\sqrt{x}(x+3)\,dx$

44. $\int(z+2)^2\,dz$

45. $\int(3u+2)^3\,du$

46. $\int\left(\frac{2}{\sqrt[5]{x}}-1\right)^2dx$

47. $\int v^{-2}(2v^4+3v^2-2v^{-3})\,dv$

48. $\int(6e^u-u^3(\sqrt{u}+1))\,du$

*49. $\int\frac{z^4+10z^3}{2z^2}\,dz$

50. $\int\frac{x^4-5x^2+2x}{5x^2}\,dx$

51. $\int\frac{e^x+e^{2x}}{e^x}\,dx$

52. $\int\frac{(x^3+1)^2}{x^2}\,dx$

53. If $F(x)$ and $G(x)$ are such that $F'(x)=G'(x)$, is it true that $F(x)-G(x)$ must be zero?

54. **(a)** Find a function F such that $\int F(x)\,dx = xe^x+C$.
(b) Is there only one function F satisfying the equation given in part (a), or are there many such functions?

55. Find $\int\frac{d}{dx}\left(\frac{1}{\sqrt{x^2+1}}\right)dx$.

OBJECTIVE

To find a particular antiderivative of a function that satisfies certain conditions. This involves evaluating constants of integration.

14.3 Integration with Initial Conditions

If we know the rate of change, f', of the function f, then the function f itself is an antiderivative of f' (since the derivative of f is f'). Of course, there are many antiderivatives of f', and the most general one is denoted by the indefinite integral. For example, if

$$f'(x) = 2x$$

then

$$f(x) = \int f'(x)\,dx = \int 2x\,dx = x^2+C. \tag{1}$$

That is, *any* function of the form $f(x) = x^2+C$ has its derivative equal to $2x$. Because of the constant of integration, notice that we do not know $f(x)$ specifically. However, if f must assume a certain function value for a particular value of x, then we can determine the value of C and thus determine $f(x)$ specifically. For instance, if $f(1)=4$, then from Equation (1),

$$f(1) = 1^2+C$$

$$4 = 1+C$$

$$C = 3$$

Thus,

$$f(x) = x^2+3$$

That is, we now know the particular function $f(x)$ for which $f'(x)=2x$ and $f(1)=4$. The condition $f(1)=4$, which gives a function value of f for a specific value of x, is called an *initial condition*.