78. $\int \frac{2}{t^2}\sqrt{\frac{1}{t}+9}\,dt = -2\int\left(\frac{1}{t}+9\right)^{\frac{1}{2}}\left[-\frac{1}{t^2}dt\right]$

$= -2\frac{\left(\frac{1}{t}+9\right)^{\frac{3}{2}}}{\frac{3}{2}}+C$

$= -\frac{4}{3}\left(\frac{1}{t}+9\right)^{\frac{3}{2}}+C$

79. Let $u=\ln\left(x^2+2x\right) \Rightarrow du = \frac{1}{x^2+2x}(2x+2)dx$

$\int \frac{x+1}{x^2+2x}\ln\left(x^2+2x\right)dx$

$=\frac{1}{2}\int \ln\left(x^2+2x\right)\left[\frac{2x+2}{x^2+2x}dx\right]$

$=\frac{1}{2}\int u\,du = \frac{1}{2}\cdot\frac{u^2}{2}+C = \frac{1}{4}\ln^2\left(x^2+2x\right)+C$

80. Let $u=\sqrt[3]{8x^4}=2x^{\frac{4}{3}} \Rightarrow du=\frac{8}{3}x^{\frac{1}{3}}dx$

$\int\sqrt[3]{x}e^{\sqrt[3]{8x^4}}dx = \frac{3}{8}\int e^{2x^{\frac{4}{3}}}\left[\frac{8}{3}x^{\frac{1}{3}}dx\right]=\frac{3}{8}\int e^u du$

$=\frac{3}{8}e^u + C = \frac{3}{8}e^{\sqrt[3]{8x^4}}+C$

81. $y=\int(3-2x)^2dx = -\frac{1}{2}\int(3-2x)^2[-2\,dx]$

$=-\frac{1}{2}\cdot\frac{(3-2x)^3}{3}+C=-\frac{1}{6}(3-2x)^3+C$

$y(0)=1$ implies $1=-\frac{1}{6}(27)+C$, so $C=\frac{11}{2}$.

Thus $y=-\frac{1}{6}(3-2x)^3+\frac{11}{2}$.

82. $y=\frac{1}{2}\int\frac{1}{x^2+6}[2x\,dx]=\frac{1}{2}\ln\left(x^2+6\right)+C$

$y(1)=0$ implies $0=\frac{1}{2}\ln(7)+C$, so $C=-\frac{1}{2}\ln 7$.

Thus $y=\frac{1}{2}\left[\ln\left(x^2+6\right)-\ln 7\right]$, or

$y=\ln\sqrt{\frac{x^2+6}{7}}$

83. $y''=\frac{1}{x^2}$

$y'=\int x^{-2}dx = -x^{-1}+C_1$

$y'(-2)=3$ implies $3=\frac{1}{2}+C_1$, so $C_1=\frac{5}{2}$. Thus

$y'=-x^{-1}+\frac{5}{2}$.

$y=\int\left(-x^{-1}+\frac{5}{2}\right)dx = -\int\frac{1}{x}dx+\int\frac{5}{2}dx$

$=-\ln|x|+\frac{5}{2}x+C_2$

$y(1)=2$ implies that $2=0+\frac{5}{2}+C_2$, so

$C_2=-\frac{1}{2}$. Thus

$y=-\ln|x|+\frac{5}{2}x-\frac{1}{2}=\ln\left|\frac{1}{x}\right|+\frac{5}{2}x-\frac{1}{2}$.

84. $y''=(x+1)^{3/2}$

$y'=\int(x+1)^{\frac{3}{2}}dx=\frac{2}{5}(x+1)^{\frac{5}{2}}+C_1$

$y'(3)=0\Rightarrow 0=\frac{2}{5}\cdot32+C_1\Rightarrow C_1=-\frac{64}{5}$, so

$y'=\frac{2}{5}(x+1)^{\frac{5}{2}}-\frac{64}{5}$

$y=\int\left[\frac{2}{5}(x+1)^{\frac{5}{2}}-\frac{64}{5}\right]dx$

$=\frac{2}{5}\cdot\frac{(x+1)^{\frac{7}{2}}}{\frac{7}{2}}-\frac{64}{5}x+C_2$

$=\frac{4}{35}(x+1)^{\frac{7}{2}}-\frac{64}{5}x+C_2$

$y(3)=0$ implies $0=\frac{4}{35}\cdot128-\frac{64}{5}(3)+C_2$, so

$C_2=\frac{832}{35}$. Thus $y=\frac{4}{35}(x+1)^{\frac{7}{2}}-\frac{64}{5}x+\frac{832}{35}$.

85. $V(t)=\int\frac{dV}{dt}dt=\int 8e^{0.05t}dt$

$=\frac{8}{0.05}\int e^{0.05t}[0.05\,dt]$

$=160e^{0.05t}+C$

The house cost $350,000 to build, so $V(0) = 350$.

$350=160e^0+C=160+C$

$190=C$

$V(t)=160e^{0.05t}+190$

86. $I(t)=\int\frac{dI}{dt}dt=\int\frac{12}{2t+50}dt$

$=6\ln|2t+50|+C$

Since the expected life span was 63 years in 1940, $I(0)=63$.

$63=6\ln|50|+C$

$C=63-6\ln 50\approx 39.53$

$I(t)=6\ln|2t+50|+39.53$

$I(58)=6\ln|166|+39.53\approx 70.20$

The expected life span for people born in 1998 (58 years after 1940) is about 70 years.

87. Note that $r > 0$.

$C=\int\left[\frac{Rr}{2K}+\frac{B_1}{r}\right]dr=\int\frac{Rr}{2K}dr+\int\frac{B_1}{r}dr$

$=\frac{R}{2K}\int r\,dr+B_1\int\frac{1}{r}dr$

$=\frac{R}{2K}\cdot\frac{r^2}{2}+B_1\ln|r|+B_2$

Thus we obtain $C=\frac{Rr^2}{4K}+B_1\ln|r|+B_2$.

88. $f(x)=\int(e^{3x+2}-3x)dx=\frac{1}{3}e^{3x+2}-\frac{3}{2}x^2+C$

$f\left(\frac{1}{3}\right)=2$ implies $2=\frac{1}{3}e^3-\frac{1}{6}+C$, so

$C=\frac{13}{6}-\frac{1}{3}e^3$. Thus,

$f(x)=\frac{1}{3}e^{3x+2}-\frac{3}{2}x^2+\frac{13}{6}-\frac{1}{3}e^3$,

$f(2)=\frac{1}{3}e^8-6+\frac{13}{6}-\frac{1}{3}e^3$

$=\frac{1}{6}(2e^8-2e^3-23)\approx 983.12$

Problems 14.5

1. $\int\frac{2x^6+8x^4-4x}{2x^2}dx$

$=\int\left(\frac{2x^6}{2x^2}+\frac{8x^4}{2x^2}-\frac{4x}{2x^2}\right)dx$

$=\int x^4dx+4\int x^2dx-2\int\frac{1}{x}dx$

$=\frac{x^5}{5}+\frac{4}{3}x^3-2\ln|x|+C$

2. $\int\frac{9x^2+5}{3x}dx=\int\left(3x+\frac{5}{3x}\right)dx$

$=\frac{3}{2}x^2+\frac{5}{3}\ln|x|+C$

3. $\int\left(3x^2+2\right)\sqrt{2x^3+4x+1}\,dx$

$=\frac{1}{2}\int\left(2x^3+4x+1\right)^{\frac{1}{2}}\left[\left(6x^2+4\right)dx\right]$

$=\frac{1}{2}\cdot\frac{\left(2x^3+4x+1\right)^{\frac{3}{2}}}{\frac{3}{2}}+C$

$=\frac{1}{3}\left(2x^3+4x+1\right)^{\frac{3}{2}}+C$

4. $\int\frac{x}{\sqrt[4]{x^2+1}}dx=\frac{1}{2}\int\left(x^2+1\right)^{-\frac{1}{4}}[2x\,dx]$

$=\frac{1}{2}\cdot\frac{\left(x^2+1\right)^{\frac{3}{4}}}{\frac{3}{4}}+C$

$=\frac{2}{3}\left(x^2+1\right)^{\frac{3}{4}}+C$

5. $\int\frac{9}{\sqrt{2-3x}}dx=9\int(2-3x)^{-1/2}dx$

$=9\left(-\frac{1}{3}\right)\int(2-3x)^{-1/2}[-3\,dx]$

$=-3\frac{(2-3x)^{1/2}}{\frac{1}{2}}+C=-6\sqrt{2-3x}+C$

6. $\int\frac{2xe^{x^2}}{e^{x^2}-2}dx=\int\frac{1}{e^{x^2}-2}\left[2xe^{x^2}dx\right]$

$=\ln\left|e^{x^2}-2\right|+C$