31. $\int \frac{4}{x}\,dx = 4\int \frac{1}{x}\,dx = 4\ln|x| + C$

32. $\int \frac{3}{1+2y}\,dy = 3\cdot\frac{1}{2}\int \frac{1}{1+2y}[2\,dy]$

$= \frac{3}{2}\ln|1+2y| + C$

33. Let $u = s^3+5 \Rightarrow du = 3s^2\,ds$

$\int \frac{s^2}{s^3+5}\,ds = \frac{1}{3}\int \frac{1}{s^3+5}\left[3s^2\,ds\right]$

$= \frac{1}{3}\int \frac{1}{u}\,du = \frac{1}{3}\ln|u| + C = \frac{1}{3}\ln\left|s^3+5\right| + C$

34. $\int \frac{2x^2}{3-4x^3}\,dx = 2\left(-\frac{1}{12}\right)\int \frac{1}{3-4x^3}\left[-12x^2\,dx\right]$

$= -\frac{1}{6}\ln\left|3-4x^3\right| + C$

35. Let $u = 4-2x \Rightarrow du = -2\,dx$

$\int \frac{5}{4-2x}\,dx = -\frac{5}{2}\int \frac{1}{4-2x}[-2\,dx]$

$= -\frac{5}{2}\int \frac{1}{u}\,du = -\frac{5}{2}\ln|u| + C = -\frac{5}{2}\ln|4-2x| + C$

36. $\int \frac{7t}{5t^2-6}\,dt = 7\cdot\frac{1}{10}\int \frac{1}{5t^2-6}[10t\,dt]$

$= \frac{7}{10}\ln\left|5t^2-6\right| + C$

37. $\int \sqrt{5x}\,dx = \sqrt{5}\int x^{1/2}\,dx = \sqrt{5}\,\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + C$

$= \frac{2\sqrt{5}}{3}x^{\frac{3}{2}} + C$

38. $\int \frac{1}{(3x)^6}\,dx = \frac{1}{3}\int (3x)^{-6}[3\,dx]$

$= \frac{1}{3}\cdot\frac{(3x)^{-5}}{-5} + C$

$= -\frac{1}{15}(3x)^{-5} + C$

39. Let $u = x^2-4 \Rightarrow du = 2x\,dx$

$\int \frac{x}{\sqrt{x^2-4}}\,dx = \frac{1}{2}\int \left(x^2-4\right)^{-\frac{1}{2}}[2x\,dx]$

$= \frac{1}{2}\int u^{-\frac{1}{2}}\,du = \frac{1}{2}\cdot\frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C$

$= \sqrt{x^2-4} + C$

40. Let $u = 1-3x \Rightarrow du = -3\,dx$.

$\int \frac{9}{1-3x}\,dx = -3\int \frac{1}{1-3x}[-3\,dx]$

$= -3\int \frac{1}{u}\,du = -3\ln|u| + C = -3\ln|1-3x| + C$

41. Let $u = y^4+1 \Rightarrow du = 4y^3\,dy$

$\int 2y^3e^{y^4+1}\,dy = 2\int y^3e^{y^4+1}\,dy$

$= 2\cdot\frac{1}{4}\int e^{y^4+1}\left[4y^3\,dy\right]$

$= \frac{1}{2}\int e^u\,du = \frac{1}{2}e^u + C$

$= \frac{1}{2}e^{y^4+1} + C$

42. $\int 2\sqrt{2x-1}\,dx = \int (2x-1)^{\frac{1}{2}}[2\,dx]$

$= \frac{(2x-1)^{\frac{3}{2}}}{\frac{3}{2}} + C$

$= \frac{2}{3}(2x-1)^{\frac{3}{2}} + C$

43. Let $u = -2v^3+1 \Rightarrow du = -6v^2\,dv$

$\int v^2e^{-2v^3+1}\,dv = -\frac{1}{6}\int e^{-2v^3+1}\left[-6v^2\,dv\right]$

$= -\frac{1}{6}\int e^u\,du = -\frac{1}{6}e^u + C$

$= -\frac{1}{6}e^{-2v^3+1} + C$

44. $\int \frac{x^2}{\sqrt[3]{2x^3+9}}\,dx = \frac{1}{6}\int \left(2x^3+9\right)^{-\frac{1}{3}}\left[6x^2\,dx\right]$

$= \frac{1}{6}\cdot\frac{\left(2x^3+9\right)^{\frac{2}{3}}}{\frac{2}{3}} + C$

$= \frac{1}{4}\left(2x^3+9\right)^{\frac{2}{3}} + C$

45. $\int \left(e^{-5x}+2e^x\right)dx = \int e^{-5x}\,dx + 2\int e^x\,dx$

$= -\frac{1}{5}\int e^{-5x}[-5\,dx] + 2\int e^x\,dx$

$= -\frac{1}{5}e^{-5x} + 2e^x + C$

46. $\int 4\sqrt[3]{y+1}\,dy = 4\int (y+1)^{\frac{1}{3}}[dy]$

$= 4\cdot\frac{(y+1)^{\frac{4}{3}}}{\frac{4}{3}} + C = 3(y+1)^{\frac{4}{3}} + C$

47. $\int (8x+10)(7-2x^2-5x)^3\,dx$

$= -2\int (7-2x^2-5x)^3[(-4x-5)dx]$

$= -2\cdot\frac{(7-2x^2-5x)^4}{4} + C$

$= -\frac{1}{2}(7-2x^2-5x)^4 + C$

48. $\int 2ye^{3y^2}\,dy = 2\cdot\frac{1}{6}\int e^{3y^2}[6y\,dy] = \frac{1}{3}e^{3y^2} + C$

49. $\int \frac{x^2+2}{x^3+6x}\,dx = \frac{1}{3}\int \frac{1}{x^3+6x}\left[\left(3x^2+6\right)dx\right]$

$= \frac{1}{3}\ln\left|x^3+6x\right| + C$

50. $\int (e^x+2e^{-3x}-e^{5x})\,dx$

$= \int e^x\,dx - \frac{2}{3}\int e^{-3x}[(-3)dx] - \frac{1}{5}\int e^{5x}[5\,dx]$

$= e^x - \frac{2}{3}e^{-3x} - \frac{1}{5}e^{5x} + C$

51. $\int \frac{16s-4}{3-2s+4s^2}\,dx = 2\int \frac{1}{3-2s+4s^2}[(8s-2)ds]$

$= 2\ln\left|3-2s+4s^2\right| + C$

52. $\int (6t^2+4t)(t^3+t^2+1)^6\,dt$

$= 2\int (t^3+t^2+1)^6[(3t^2+2t)dt]$

$= 2\cdot\frac{(t^3+t^2+1)^7}{7} + C$

$= \frac{2}{7}(t^3+t^2+1)^7 + C$

53. $\int x\left(2x^2+1\right)^{-1}\,dx = \int \frac{x}{2x^2+1}\,dx$

$= \frac{1}{4}\int \frac{1}{2x^2+1}[4x\,dx]$

$= \frac{1}{4}\ln\left(2x^2+1\right) + C$

54. $\int (8w^5+w^2-2)(6w-w^3-4w^6)^{-4}\,dw$

$= -\frac{1}{3}\int (6w-w^3-4w^6)^{-4}[(6-3w^2-24w^5)dw]$

$= -\frac{1}{3}\cdot\frac{(6w-w^3-4w^6)^{-3}}{-3} + C$

$= \frac{1}{9}(6w-w^3-4w^6)^{-3} + C$

55. $\int -\left(x^2-2x^5\right)\left(x^3-x^6\right)^{-10}\,dx$

$= -\frac{1}{3}\int \left(x^3-x^6\right)^{-10}\left[\left(3x^2-6x^5\right)dx\right]$

$= -\frac{1}{3}\cdot\frac{\left(x^3-x^6\right)^{-9}}{-9} + C = \frac{1}{27}\left(x^3-x^6\right)^{-9} + C$

56. $\int \frac{3}{5}(v-2)e^{2-4v+v^2}\,dv$

$= \frac{3}{5}\cdot\frac{1}{2}\int e^{2-4v+v^2}[(2v-4)\,dv]$

$= \frac{3}{10}e^{2-4v+v^2} + C$

57. $\int \left(2x^3+x\right)\left(x^4+x^2\right)dx$

$= \frac{1}{2}\int \left(x^4+x^2\right)^1\left[\left(4x^3+2x\right)dx\right]$

$= \frac{1}{2}\cdot\frac{\left(x^4+x^2\right)^2}{2} + C = \frac{1}{4}\left(x^4+x^2\right)^2 + C$