

5. Let $u = y^3 + 3y^2 + 1 \Rightarrow du = (3y^2 + 6y) dy$

$$\begin{aligned} & \int (3y^2 + 6y)(y^3 + 3y^2 + 1)^{\frac{2}{3}} dy \\ &= \int (y^3 + 3y^2 + 1)^{\frac{2}{3}} [(3y^2 + 6y) dy] \\ &= \int u^{\frac{2}{3}} du = \frac{u^{\frac{5}{3}}}{\frac{5}{3}} + C \\ &= \frac{3}{5} (y^3 + 3y^2 + 1)^{\frac{5}{3}} + C \end{aligned}$$

6. $\int (15t^2 - 6t + 1)(5t^3 - 3t^2 + t)^{17} dt$
 $= \int (5t^3 - 3t^2 + t)^{17} [(15t^2 - 6t + 1) dt]$
 $= \frac{(5t^3 - 3t^2 + t)^{18}}{18} + C$

7. Let $u = 3x - 1 \Rightarrow du = 3 dx$
 $\int \frac{5}{(3x-1)^3} dx = \frac{5}{3} \int \frac{1}{(3x-1)^3} [3 dx]$
 $= \frac{5}{3} \int u^{-3} du = \frac{5}{3} \int u^{-3} du$
 $= \frac{5}{3} \cdot \frac{u^{-2}}{-2} + C = -\frac{5(3x-1)^{-2}}{6} + C$

8. $\int \frac{4x}{(2x^2 - 7)^{10}} dx = \int (2x^2 - 7)^{-10} [4x dx]$
 $= -\frac{(2x^2 - 7)^{-9}}{9} + C$

9. Let $u = 2x - 1 \Rightarrow du = 2 dx$
 $\int \sqrt{2x-1} dx = \int (2x-1)^{\frac{1}{2}} dx$
 $= \frac{1}{2} \int (2x-1)^{\frac{1}{2}} [2 dx]$
 $= \frac{1}{2} \int u^{\frac{1}{2}} du = \frac{1}{2} \cdot \frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{1}{3} (2x-1)^{\frac{3}{2}} + C$

10. Let $u = x - 5 \Rightarrow du = dx$
 $\int \frac{1}{\sqrt{x-5}} dx = \int (x-5)^{-\frac{1}{2}} [dx]$
 $\int u^{-1/2} du = \frac{u^{1/2}}{\frac{1}{2}} + C$
 $= \frac{(x-5)^{\frac{1}{2}}}{\frac{1}{2}} + C = 2\sqrt{x-5} + C$

11. Let $u = 7x - 6 \Rightarrow du = 7 dx$
 $\int (7x-6)^4 dx = \frac{1}{7} \int (7x-6)^4 [7 dx]$
 $= \frac{1}{7} \int u^4 du = \frac{1}{7} \cdot \frac{u^5}{5} + C$
 $= \frac{(7x-6)^5}{35} + C$

12. $\int x^2 (3x^3 + 7)^3 dx = \frac{1}{9} \int (3x^3 + 7)^3 [9x^2 dx]$
 $= \frac{1}{9} \cdot \frac{(3x^3 + 7)^4}{4} + C$
 $= \frac{(3x^3 + 7)^4}{36} + C$

13. Let $v = 5u^2 - 9 \Rightarrow dv = 10u du$
 $\int u(5u^2 - 9)^{14} du = \frac{1}{10} \int (5u^2 - 9)^{14} [10u du]$
 $\frac{1}{10} \int v^{14} dv = \frac{1}{10} \cdot \frac{v^{15}}{15} + C = \frac{(5u^2 - 9)^{15}}{150} + C$

14. $\int 9x\sqrt{1+2x^2} dx = \frac{9}{4} \int (1+2x^2)^{\frac{1}{2}} [4x dx]$
 $= \frac{9}{4} \cdot \frac{(1+2x^2)^{\frac{3}{2}}}{\frac{3}{2}} + C$
 $= \frac{3(1+2x^2)^{\frac{3}{2}}}{2} + C$

15. Let $u = 27 + x^5 \Rightarrow du = 5x^4 dx$
 $\int 4x^4 (27 + x^5)^{\frac{1}{3}} dx = \frac{4}{5} \int (27 + x^5)^{\frac{1}{3}} [5x^4 dx]$
 $= \frac{4}{5} \int u^{\frac{1}{3}} du = \frac{4}{5} \cdot \frac{u^{\frac{4}{3}}}{\frac{4}{3}} + C$
 $= \frac{3}{5} (27 + x^5)^{\frac{4}{3}} + C$

16. Let $u = 4 - 5x \Rightarrow du = -5 dx$
 $\int (4-5x)^9 dx = -\frac{1}{5} \int (4-5x)^9 [-5 dx]$
 $= -\frac{1}{5} \int u^9 du = -\frac{1}{5} \cdot \frac{u^{10}}{10} + C = -\frac{1}{50} (4-5x)^{10} + C$

17. Let $u = 3x \Rightarrow du = 3 dx$
 $\int 3e^{3x} dx = \int e^{3x} [3 dx]$
 $= \int e^u du = e^u + C = e^{3x} + C$

18. $\int 5e^{3t+7} dt = \frac{5}{3} \int e^{3t+7} [3 dt] = \frac{5}{3} e^{3t+7} + C$

19. Let $u = t^2 + t \Rightarrow du = (2t+1) dt$
 $\int (2t+1)e^{t^2+t} dt = \int e^{t^2+t} [(2t+1) dt]$
 $= \int e^u du = e^u + C = e^{t^2+t} + C$

20. $\int -3w^2 e^{-w^3} dw = \int e^{-w^3} [-3w^2 dw] = e^{-w^3} + C$

21. Let $u = 7x^2 \Rightarrow du = 14x dx$
 $\int xe^{7x^2} dx = \frac{1}{14} \int e^{7x^2} [14x dx] = \frac{1}{14} \int e^u du$
 $= \frac{1}{14} e^u + C = \frac{1}{14} e^{7x^2} + C$

22. $\int x^3 e^{4x^4} dx = \frac{1}{16} \int e^{4x^4} [16x^3 dx]$
 $= \frac{1}{16} \cdot e^{4x^4} + C = \frac{e^{4x^4}}{16} + C$

23. Let $u = -3x \Rightarrow du = -3 dx$
 $\int 4e^{-3x} dx = -\frac{4}{3} \int e^{-3x} [-3 dx]$
 $= -\frac{4}{3} \int e^u du = -\frac{4}{3} e^u + C = -\frac{4}{3} e^{-3x} + C$

24. $\int x^4 e^{-6x^5} dx = -\frac{1}{30} \int e^{-6x^5} [-30x^4 dx]$
 $= -\frac{1}{30} e^{-6x^5} + C$

25. Let $u = x + 5 \Rightarrow du = dx$
 $\int \frac{1}{x+5} [dx] = \int \frac{1}{u} du = \ln|u| + C = \ln|x+5| + C$

26. $\int \frac{12x^2 + 4x + 2}{x + x^2 + 2x^3} dx$
 $= \int \frac{2}{x + x^2 + 2x^3} [(1 + 2x + 6x^2) dx]$
 $= 2 \ln|x + x^2 + 2x^3| + C$
 $= \ln[(x + x^2 + 2x^3)^2] + C$

27. Let $u = x^3 + x^4 \Rightarrow du = (3x^2 + 4x^3) dx$
 $\int \frac{3x^2 + 4x^3}{x^3 + x^4} dx = \int \frac{1}{x^3 + x^4} [(3x^2 + 4x^3) dx]$
 $= \int \frac{1}{u} du = \ln|u| + C$
 $= \ln|x^3 + x^4| + C$

28. Let $u = 1 - 3x^2 + 2x^3 \Rightarrow du = (-6x + 6x^2) dx$
 $\int \frac{6x^2 - 6x}{1 - 3x^2 + 2x^3} dx$
 $= \int \frac{1}{1 - 3x^2 + 2x^3} [(-6x + 6x^2) dx]$
 $= \int \frac{1}{u} du = \ln|u| + C = \ln|1 - 3x^2 + 2x^3| + C$

29. Let $u = z^2 - 6 \Rightarrow du = 2z dz$
 $\int \frac{6z}{(z^2 - 6)^5} dz = 3 \int (z^2 - 6)^{-5} [2z dz]$
 $= 3 \int u^{-5} du = 3 \frac{u^{-4}}{-4} + C = -\frac{3}{4} (z^2 - 6)^{-4} + C$

30. $\int \frac{3}{(5v-1)^4} dv = \frac{3}{5} \int (5v-1)^{-4} [5 dv]$
 $= \frac{3}{5} \cdot \frac{(5v-1)^{-3}}{-3} + C$
 $= -\frac{1}{5} (5v-1)^{-3} + C$