5. Let $u = y^3 + 3y^2 + 1 \Rightarrow du = \left(3y^2 + 6y\right)dy$

$$\int\left(3y^2+6y\right)\left(y^3+3y^2+1\right)^{\frac{2}{3}}dy$$
$$= \int\left(y^3+3y^2+1\right)^{\frac{2}{3}}\left[\left(3y^2+6y\right)dy\right]$$
$$= \int u^{\frac{2}{3}}du = \frac{u^{\frac{5}{3}}}{\frac{5}{3}} + C$$
$$= \frac{3}{5}\left(y^3+3y^2+1\right)^{\frac{5}{3}} + C$$

6. $\int(15t^2-6t+1)(5t^3-3t^2+t)^{17}dt$
$$= \int(5t^3-3t^2+t)^{17}[(15t^2-6t+1)dt]$$
$$= \frac{(5t^3-3t^2+t)^{18}}{18} + C$$

7. Let $u = 3x-1 \Rightarrow du = 3\,dx$
$$\int\frac{5}{(3x-1)^3}dx = \frac{5}{3}\int\frac{1}{(3x-1)^3}[3\,dx]$$
$$= \frac{5}{3}\int\frac{1}{u^3}du = \frac{5}{3}\int u^{-3}du$$
$$= \frac{5}{3}\cdot\frac{u^{-2}}{-2} + C = -\frac{5(3x-1)^{-2}}{6} + C$$

8. $\int\frac{4x}{\left(2x^2-7\right)^{10}}dx = \int\left(2x^2-7\right)^{-10}[4x\,dx]$
$$= -\frac{\left(2x^2-7\right)^{-9}}{9} + C$$

9. Let $u = 2x-1 \Rightarrow du = 2\,dx$.
$$\int\sqrt{2x-1}\,dx = \int(2x-1)^{\frac{1}{2}}dx$$
$$= \frac{1}{2}\int(2x-1)^{\frac{1}{2}}[2\,dx]$$
$$= \frac{1}{2}\int u^{\frac{1}{2}}du = \frac{1}{2}\cdot\frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{1}{3}(2x-1)^{\frac{3}{2}} + C$$

10. Let $u = x-5 \Rightarrow du = dx$.
$$\int\frac{1}{\sqrt{x-5}}dx = \int(x-5)^{-\frac{1}{2}}[dx]$$
$$\int u^{-1/2}du = \frac{u^{1/2}}{\frac{1}{2}} + C$$
$$= \frac{(x-5)^{\frac{1}{2}}}{\frac{1}{2}} + C = 2\sqrt{x-5} + C\,.$$

11. Let $u = 7x-6 \Rightarrow du = 7\,dx$
$$\int(7x-6)^4dx = \frac{1}{7}\int(7x-6)^4[7\,dx]$$
$$= \frac{1}{7}\int u^4du = \frac{1}{7}\cdot\frac{u^5}{5} + C$$
$$= \frac{(7x-6)^5}{35} + C$$

12. $\int x^2\left(3x^3+7\right)^3dx = \frac{1}{9}\int\left(3x^3+7\right)^3\left[9x^2dx\right]$
$$= \frac{1}{9}\cdot\frac{\left(3x^3+7\right)^4}{4} + C$$
$$= \frac{\left(3x^3+7\right)^4}{36} + C$$

13. Let $v = 5u^2-9 \Rightarrow dv = 10u\,du$
$$\int u(5u^2-9)^{14}du = \frac{1}{10}\int(5u^2-9)^{14}[10u\,du]$$
$$\frac{1}{10}\int v^{14}dv = \frac{1}{10}\cdot\frac{v^{15}}{15} + C = \frac{(5u^2-9)^{15}}{150} + C\,.$$

14. $\int 9x\sqrt{1+2x^2}\,dx = \frac{9}{4}\int\left(1+2x^2\right)^{\frac{1}{2}}[4x\,dx]$
$$= \frac{9}{4}\cdot\frac{\left(1+2x^2\right)^{\frac{3}{2}}}{\frac{3}{2}} + C$$
$$= \frac{3\left(1+2x^2\right)^{\frac{3}{2}}}{2} + C$$

15. Let $u = 27+x^5 \Rightarrow du = 5x^4dx$
$$\int 4x^4\left(27+x^5\right)^{\frac{1}{3}}dx = \frac{4}{5}\int\left(27+x^5\right)^{\frac{1}{3}}\left[5x^4dx\right]$$
$$= \frac{4}{5}\int u^{\frac{1}{3}}du = \frac{4}{5}\cdot\frac{u^{\frac{4}{3}}}{\frac{4}{3}} + C$$
$$= \frac{3}{5}\left(27+x^5\right)^{\frac{4}{3}} + C$$

16. Let $u = 4-5x \Rightarrow du = -5dx$.
$$\int(4-5x)^9dx = -\frac{1}{5}\int(4-5x)^9[-5\,dx]$$
$$= -\frac{1}{5}\int u^9du = -\frac{1}{5}\cdot\frac{u^{10}}{10} + C = -\frac{1}{50}(4-5x)^{10} + C$$

17. Let $u = 3x \Rightarrow du = 3\,dx$
$$\int 3e^{3x}dx = \int e^{3x}[3\,dx]$$
$$= \int e^u du = e^u + C = e^{3x} + C$$

18. $\int 5e^{3t+7}dt = \frac{5}{3}\int e^{3t+7}[3\,dt] = \frac{5}{3}e^{3t+7} + C$

19. Let $u = t^2+t \Rightarrow du = (2t+1)dt$
$$\int(2t+1)e^{t^2+t}dt = \int e^{t^2+t}[(2t+1)\,dt]$$
$$= \int e^u du = e^u + C = e^{t^2+t} + C$$

20. $\int -3w^2e^{-w^3}dw = \int e^{-w^3}\left[-3w^2dw\right] = e^{-w^3} + C$

21. Let $u = 7x^2 \Rightarrow du = 14x\,dx$
$$\int xe^{7x^2}dx = \frac{1}{14}\int e^{7x^2}[14x\,dx] = \frac{1}{14}\int e^u du$$
$$= \frac{1}{14}e^u + C = \frac{1}{14}e^{7x^2} + C$$

22. $\int x^3e^{4x^4}dx = \frac{1}{16}\int e^{4x^4}\left[16x^3dx\right]$
$$= \frac{1}{16}\cdot e^{4x^4} + C = \frac{e^{4x^4}}{16} + C$$

23. Let $u = -3x \Rightarrow du = -3dx$.
$$\int 4e^{-3x}dx = -\frac{4}{3}\int e^{-3x}[-3\,dx]$$
$$= -\frac{4}{3}\int e^u du = -\frac{4}{3}e^u + C = -\frac{4}{3}e^{-3x} + C$$

24. $\int x^4e^{-6x^5}dx = -\frac{1}{30}\int e^{-6x^5}\left[-30x^4dx\right]$
$$= -\frac{1}{30}e^{-6x^5} + C$$

25. Let $u = x+5 \Rightarrow du = dx$
$$\int\frac{1}{x+5}[dx] = \int\frac{1}{u}du = \ln|u| + C = \ln|x+5| + C$$

26. $\int\frac{12x^2+4x+2}{x+x^2+2x^3}dx$
$$= \int\frac{2}{x+x^2+2x^3}[(1+2x+6x^2)dx]$$
$$= 2\ln\left|x+x^2+2x^3\right| + C$$
$$= \ln[(x+x^2+2x^3)^2] + C$$

27. Let $u = x^3+x^4 \Rightarrow du = (3x^2+4x^3)dx$
$$\int\frac{3x^2+4x^3}{x^3+x^4}dx = \int\frac{1}{x^3+x^4}\left[\left(3x^2+4x^3\right)dx\right]$$
$$= \int\frac{1}{u}du = \ln|u| + C$$
$$= \ln\left|x^3+x^4\right| + C$$

28. Let $u = 1-3x^2+2x^3 \Rightarrow du = (-6x+6x^2)dx$.
$$\int\frac{6x^2-6x}{1-3x^2+2x^3}dx$$
$$= \int\frac{1}{1-3x^2+2x^3}[(-6x+6x^2)dx$$
$$= \int\frac{1}{u}du = \ln|u| + C = \ln\left|1-3x^2+2x^3\right| + C$$

29. Let $u = z^2-6 \Rightarrow du = 2z\,dz$
$$\int\frac{6z}{(z^2-6)^5} = 3\int(z^2-6)^{-5}[2z\,dz]$$
$$= 3\int u^{-5}du = 3\frac{u^{-4}}{-4} + C = -\frac{3}{4}(z^2-6)^{-4} + C$$

30. $\int\frac{3}{(5v-1)^4}dv = \frac{3}{5}\int(5v-1)^{-4}[5\,dv]$
$$= \frac{3}{5}\cdot\frac{(5v-1)^{-3}}{-3} + C$$
$$= -\frac{1}{5}(5v-1)^{-3} + C$$

31. $\int \frac{4}{x}dx = 4\int\frac{1}{x}dx = 4\ln|x| + C$

32. $\int \frac{3}{1+2y}dy = 3\cdot\frac{1}{2}\int\frac{1}{1+2y}[2\,dy]$
$= \frac{3}{2}\ln|1+2y|+C$

33. Let $u = s^3+5 \Rightarrow du = 3s^2\,ds$
$\int\frac{s^2}{s^3+5}ds = \frac13\int\frac{1}{s^3+5}\left[3s^2ds\right]$
$=\frac13\int\frac1u du = \frac13\ln|u|+C = \frac13\ln\left|s^3+5\right|+C$

34. $\int\frac{2x^2}{3-4x^3}dx = 2\left(-\frac{1}{12}\right)\int\frac{1}{3-4x^3}\left[-12x^2dx\right]$
$=-\frac16\ln\left|3-4x^3\right|+C$

35. Let $u=4-2x \Rightarrow du=-2\,dx$
$\int\frac{5}{4-2x}dx = -\frac52\int\frac{1}{4-2x}[-2dx]$
$=-\frac52\int\frac1u du = -\frac52\ln|u|+C = -\frac52\ln|4-2x|+C$

36. $\int\frac{7t}{5t^2-6}dt = 7\cdot\frac1{10}\int\frac1{5t^2-6}[10t\,dt]$
$=\frac7{10}\ln\left|5t^2-6\right|+C$

37. $\int\sqrt{5x}\,dx = \sqrt5\int x^{1/2}dx = \sqrt5\frac{x^{\frac32}}{\frac32}+C$
$=\frac{2\sqrt5}{3}x^{\frac32}+C$

38. $\int\frac{1}{(3x)^6}dx = \frac13\int(3x)^{-6}[3dx]$
$=\frac13\cdot\frac{(3x)^{-5}}{-5}+C$
$=-\frac1{15}(3x)^{-5}+C$

39. Let $u=x^2-4\Rightarrow du=2x\,dx$
$\int\frac{x}{\sqrt{x^2-4}}dx=\frac12\int\left(x^2-4\right)^{-\frac12}[2x\,dx]$
$=\frac12\int u^{-\frac12}du = \frac12\cdot\frac{u^{\frac12}}{\frac12}+C$
$=\sqrt{x^2-4}+C$

40. Let $u=1-3x\Rightarrow du=-3\,dx$.
$\int\frac{9}{1-3x}dx=-3\int\frac{1}{1-3x}[-3\,dx]$
$=-3\int\frac1u du=-3\ln|u|+C=-3\ln|1-3x|+C$

41. Let $u=y^4+1\Rightarrow du=4y^3dy$
$\int 2y^3e^{y^4+1}dy = 2\int y^3e^{y^4+1}dy$
$=2\cdot\frac14\int e^{y^4+1}\left[4y^3dy\right]$
$=\frac12\int e^u du=\frac12e^u+C$
$=\frac12e^{y^4+1}+C$

42. $\int2\sqrt{2x-1}\,dx=\int(2x-1)^{\frac12}[2\,dx]$
$=\frac{(2x-1)^{\frac32}}{\frac32}+C$
$=\frac23(2x-1)^{\frac32}+C$

43. Let $u=-2v^3+1\Rightarrow du=-6v^2dv$
$\int v^2e^{-2v^3+1}dv=-\frac16\int e^{-2v^3+1}\left[-6v^2dv\right]$
$=-\frac16\int e^u du=-\frac16e^u+C$
$=-\frac16e^{-2v^3+1}+C$

44. $\int\frac{x^2}{\sqrt[3]{2x^3+9}}dx=\frac16\int\left(2x^3+9\right)^{-\frac13}\left[6x^2dx\right]$
$=\frac16\cdot\frac{\left(2x^3+9\right)^{\frac23}}{\frac23}+C$
$=\frac14\left(2x^3+9\right)^{\frac23}+C$

45. $\int\left(e^{-5x}+2e^x\right)dx=\int e^{-5x}dx+2\int e^xdx$
$=-\frac15\int e^{-5x}[-5\,dx]+2\int e^xdx$
$=-\frac15e^{-5x}+2e^x+C$

46. $\int4\sqrt[3]{y+1}\,dy=4\int(y+1)^{\frac13}[dy]$
$=4\cdot\frac{(y+1)^{\frac43}}{\frac43}+C=3(y+1)^{\frac43}+C$

47. $\int(8x+10)(7-2x^2-5x)^3dx$
$=-2\int(7-2x^2-5x)^3[(-4x-5)dx]$
$=-2\cdot\frac{(7-2x^2-5x)^4}{4}+C$
$=-\frac12(7-2x^2-5x)^4+C$

48. $\int2ye^{3y^2}dy=2\cdot\frac16\int e^{3y^2}[6y\,dy]=\frac13e^{3y^2}+C$

49. $\int\frac{x^2+2}{x^3+6x}dx=\frac13\int\frac1{x^3+6x}\left[\left(3x^2+6\right)dx\right]$
$=\frac13\ln\left|x^3+6x\right|+C$

50. $\int(e^x+2e^{-3x}-e^{5x})dx$
$=\int e^x\,dx-\frac23\int e^{-3x}[(-3)dx]-\frac15\int e^{5x}[5\,dx]$
$=e^x-\frac23e^{-3x}-\frac15e^{5x}+C$

51. $\int\frac{16s-4}{3-2s+4s^2}ds=2\int\frac{1}{3-2s+4s^2}[(8s-2)ds]$
$=2\ln\left|3-2s+4s^2\right|+C$

52. $\int(6t^2+4t)(t^3+t^2+1)^6dt$
$=2\int(t^3+t^2+1)^6[(3t^2+2t)dt]$
$=2\cdot\frac{(t^3+t^2+1)^7}{7}+C$
$=\frac27(t^3+t^2+1)^7+C$

53. $\int x\left(2x^2+1\right)^{-1}dx=\int\frac{x}{2x^2+1}dx$
$=\frac14\int\frac1{2x^2+1}[4x\,dx]$
$=\frac14\ln\left(2x^2+1\right)+C$

54. $\int(8w^5+w^2-2)(6w-w^3-4w^6)^{-4}dw$
$=-\frac13\int(6w-w^3-4w^6)^{-4}[(6-3w^2-24w^5)dw]$
$=-\frac13\cdot\frac{(6w-w^3-4w^6)^{-3}}{-3}+C$
$=\frac19(6w-w^3-4w^6)^{-3}+C$

55. $\int-\left(x^2-2x^5\right)\left(x^3-x^6\right)^{-10}dx$
$=-\frac13\int\left(x^3-x^6\right)^{-10}\left[\left(3x^2-6x^5\right)dx\right]$
$=-\frac13\cdot\frac{\left(x^3-x^6\right)^{-9}}{-9}+C=\frac1{27}\left(x^3-x^6\right)^{-9}+C$

56. $\int\frac35(v-2)e^{2-4v+v^2}dv$
$=\frac35\cdot\frac12\int e^{2-4v+v^2}[(2v-4)\,dv]$
$=\frac3{10}e^{2-4v+v^2}+C$

57. $\int\left(2x^3+x\right)\left(x^4+x^2\right)dx$
$=\frac12\int\left(x^4+x^2\right)^1\left[\left(4x^3+2x\right)dx\right]$
$=\frac12\cdot\frac{\left(x^4+x^2\right)^2}{2}+C=\frac14\left(x^4+x^2\right)^2+C$

58. $\int\left(e^{3.1}\right)^2 dx = \int e^{6.2}dx = e^{6.2}x + C$, because $e^{6.2}$ is a constant.

59. $\int \frac{7+14x}{(4-x-x^2)^5}dx$

$= -7\int (4-x-x^2)^{-5}[(-1-2x)dx]$

$= -7\frac{(4-x-x^2)^{-4}}{-4} + C$

$= \frac{7}{4}(4-x-x^2)^{-4} + C$

60. $\int\left(e^x - e^{-x}\right)^2 dx = \int\left(e^{2x} - 2 + e^{-2x}\right) dx$

$= \frac{1}{2}\int e^{2x}[2\ dx] - \int 2\ dx + \left(-\frac{1}{2}\right)\int e^{-2x}[-2\ dx]$

$= \frac{1}{2}e^{2x} - 2x - \frac{1}{2}e^{-2x} + C$

$= \frac{1}{2}\left(e^{2x} - e^{-2x}\right) - 2x + C$

61. $u = 4x^3 + 3x^2 - 4$

$du = \left(12x^2 + 6x\right)dx = 6x(2x+1)dx$

$\int x(2x+1)e^{4x^3+3x^2-4}dx$

$= \frac{1}{6}\int e^{4x^3+3x^2-4}[6x(2x+1)dx]$

$= \frac{1}{6}\int e^u du = \frac{1}{6}e^u + C = \frac{1}{6}e^{4x^3+3x^2-4} + C$

62. $\int(u^3 - ue^{6-3u^2})du = \frac{u^4}{4} + \frac{1}{6}\int e^{6-3u^2}[-6u\ du]$

$= \frac{1}{4}u^4 + \frac{1}{6}e^{6-3u^2} + C$

63. $\int x\sqrt{\left(8-5x^2\right)^3}dx = -\frac{1}{10}\int\left(8-5x^2\right)^{\frac{3}{2}}[-10x\ dx]$

$= -\frac{1}{10}\cdot\frac{\left(8-5x^2\right)^{\frac{5}{2}}}{\frac{5}{2}} + C = -\frac{1}{25}\left(8-5x^2\right)^{\frac{5}{2}} + C$

64. $\int e^{-\frac{x}{7}}dx = -7\int e^{-\frac{x}{7}}\left[-\frac{1}{7}dx\right] = -7e^{-\frac{x}{7}} + C$

65. $\int\left(\sqrt{2x} - \frac{1}{\sqrt{2x}}\right)dx = \int\sqrt{2x}dx - \int\frac{1}{\sqrt{2x}}dx$

$= \frac{1}{2}\int(2x)^{\frac{1}{2}}[2\ dx] - \frac{1}{2}\int(2x)^{-\frac{1}{2}}[2\ dx]$

$= \frac{1}{2}\cdot\frac{(2x)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{1}{2}\cdot\frac{(2x)^{\frac{1}{2}}}{\frac{1}{2}} + C = \frac{(2x)^{\frac{3}{2}}}{3} - \sqrt{2x} + C$

$= \frac{2\sqrt{2}}{3}x^{\frac{3}{2}} - \sqrt{2}x^{\frac{1}{2}} + C$

66. $\int 3\frac{x^4}{e^{x^5}}dx = 3\int x^4e^{-x^5}dx = -\frac{3}{5}\int e^{-x^5}[-5x^4\ dx]$

$= -\frac{3}{5}e^{-x^5} + C$

67. $\int\left(x^2+1\right)^2 dx = \int\left(x^4 + 2x^2 + 1\right)dx$

$= \frac{x^5}{5} + \frac{2x^3}{3} + x + C$

68. $\int\left[x\left(x^2-16\right)^2 - \frac{1}{2x+5}\right]dx$

$= \frac{1}{2}\int\left(x^2-16\right)^2[2x\ dx] - \frac{1}{2}\int\frac{1}{2x+5}[2\ dx]$

$= \frac{1}{2}\cdot\frac{\left(x^2-16\right)^3}{3} - \frac{1}{2}\ln|2x+5| + C$

$= \frac{1}{6}\left(x^2-16\right)^3 - \frac{1}{2}\ln|2x+5| + C$

69. $\int\left[\frac{x}{x^2+1} + \frac{x^5}{(x^6+1)^2}\right]dx$

$= \int\frac{x}{x^2+1}dx + \int\frac{x^5}{\left(x^6+1\right)^2}dx$

$= \frac{1}{2}\int\frac{1}{x^2+1}[2x\ dx] + \frac{1}{6}\int\left(x^6+1\right)^{-2}\left[6x^5dx\right]$

$= \frac{1}{2}\ln\left(x^2+1\right) + \frac{1}{6}\cdot\frac{\left(x^6+1\right)^{-1}}{-1} + C$

$= \frac{1}{2}\ln\left(x^2+1\right) - \frac{1}{6\left(x^6+1\right)} + C$

70. $\int\left[\frac{3}{x-1} + \frac{1}{(x-1)^2}\right]dx = \int\frac{3}{x-1}[dx] + \int(x-1)^{-2}[dx]$

$= 3\ln|x-1| + \frac{(x-1)^{-1}}{-1} + C = 3\ln|x-1| - \frac{1}{x-1} + C$

71. $\int\left[\frac{2}{4x+1} - (4x^2 - 8x^5)(x^3 - x^6)^{-8}\right]dx$

$= \frac{1}{2}\int\frac{1}{4x+1}[4\,dx] - \frac{4}{3}\int(x^3 - x^6)^{-8}[(3x^2 - 6x^5)dx]$

$= \frac{1}{2}\ln|4x+1| - \frac{4}{3}\cdot\frac{(x^3-x^6)^{-7}}{-7} + C$

$= \frac{1}{2}\ln|4x+1| + \frac{4}{21}(x^3 - x^6)^{-7} + C$

72. $\int\left(r^3+5\right)^2 dr = \int\left(r^6 + 10r^3 + 25\right)dr = \frac{1}{7}r^7 + \frac{5}{2}r^4 + 25r + C$

73. $\int\left[\sqrt{3x+1} - \frac{x}{x^2+3}\right]dx = \int(3x+1)^{\frac{1}{2}}dx - \int\frac{x}{x^2+3}dx = \frac{1}{3}\int(3x+1)^{\frac{1}{2}}[3\ dx] - \frac{1}{2}\int\frac{1}{x^2+3}[2x\ dx]$

$= \frac{1}{3}\cdot\frac{(3x+1)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{1}{2}\ln\left(x^2+3\right) + C = \frac{2}{9}(3x+1)^{\frac{3}{2}} - \ln\sqrt{x^2+3} + C$

74. $\int\left[\frac{x}{3x^2+5} - \frac{x^2}{(x^3+1)^3}\right]dx = \frac{1}{6}\int\frac{1}{3x^2+5}[6x\,dx] - \frac{1}{3}\int(x^3+1)^{-3}[3x^2\ dx]$

$= \frac{1}{6}\ln\left|3x^2+5\right| - \frac{1}{3}\cdot\frac{(x^3+1)^{-2}}{-2} + C = \frac{1}{6}\ln\left|3x^2+5\right| + \frac{1}{6}(x^3+1)^{-2} + C$

75. Let $u = \sqrt{x} \Rightarrow du = \frac{1}{2}x^{-\frac{1}{2}}dx = \frac{1}{2\sqrt{x}}dx$.

$\int\frac{e^{\sqrt{x}}}{\sqrt{x}}dx = 2\int e^{\sqrt{x}}\left[\frac{1}{2\sqrt{x}}dx\right]$

$= 2\int e^u du = 2e^u + C = 2e^{\sqrt{x}} + C$

76. $\int\left(e^5 - 3^e\right)dx = \left(e^5 - 3^e\right)x + C$, because $e^5 - 3^e$ is a constant.

77. $\int\frac{1+e^{2x}}{4e^x}dx = \frac{1}{4}\int\left(\frac{1}{e^x} + \frac{e^{2x}}{e^x}\right)dx$

$= \frac{1}{4}\int\left(e^{-x} + e^x\right)dx$

$= -\frac{1}{4}\int e^{-x}[-1\ dx] + \frac{1}{4}\int e^x dx$

$= -\frac{1}{4}e^{-x} + \frac{1}{4}e^x + C$

78. $\int \frac{2}{t^2}\sqrt{\frac{1}{t}+9}\,dt = -2\int\left(\frac{1}{t}+9\right)^{\frac{1}{2}}\left[-\frac{1}{t^2}dt\right]$

$= -2\frac{\left(\frac{1}{t}+9\right)^{\frac{3}{2}}}{\frac{3}{2}}+C$

$= -\frac{4}{3}\left(\frac{1}{t}+9\right)^{\frac{3}{2}}+C$

79. Let $u=\ln\left(x^2+2x\right) \Rightarrow du = \frac{1}{x^2+2x}(2x+2)dx$

$\int \frac{x+1}{x^2+2x}\ln\left(x^2+2x\right)dx$

$=\frac{1}{2}\int \ln\left(x^2+2x\right)\left[\frac{2x+2}{x^2+2x}dx\right]$

$=\frac{1}{2}\int u\,du = \frac{1}{2}\cdot\frac{u^2}{2}+C=\frac{1}{4}\ln^2\left(x^2+2x\right)+C$

80. Let $u=\sqrt[3]{8x^4}=2x^{\frac{4}{3}}\Rightarrow du=\frac{8}{3}x^{\frac{1}{3}}dx$

$\int\sqrt[3]{x}e^{\sqrt[3]{8x^4}}dx=\frac{3}{8}\int e^{2x^{\frac{4}{3}}}\left[\frac{8}{3}x^{\frac{1}{3}}dx\right]=\frac{3}{8}\int e^u du$

$=\frac{3}{8}e^u+C=\frac{3}{8}e^{\sqrt[3]{8x^4}}+C$

81. $y=\int(3-2x)^2dx=-\frac{1}{2}\int(3-2x)^2[-2\,dx]$

$=-\frac{1}{2}\cdot\frac{(3-2x)^3}{3}+C=-\frac{1}{6}(3-2x)^3+C$

$y(0)=1$ implies $1=-\frac{1}{6}(27)+C$, so $C=\frac{11}{2}$.

Thus $y=-\frac{1}{6}(3-2x)^3+\frac{11}{2}$.

82. $y=\frac{1}{2}\int\frac{1}{x^2+6}[2x\,dx]=\frac{1}{2}\ln\left(x^2+6\right)+C$

$y(1)=0$ implies $0=\frac{1}{2}\ln(7)+C$, so $C=-\frac{1}{2}\ln 7$.

Thus $y=\frac{1}{2}\left[\ln\left(x^2+6\right)-\ln 7\right]$, or

$y=\ln\sqrt{\frac{x^2+6}{7}}$

83. $y''=\frac{1}{x^2}$

$y'=\int x^{-2}dx=-x^{-1}+C_1$

$y'(-2)=3$ implies $3=\frac{1}{2}+C_1$, so $C_1=\frac{5}{2}$. Thus

$y'=-x^{-1}+\frac{5}{2}$.

$y=\int\left(-x^{-1}+\frac{5}{2}\right)dx=-\int\frac{1}{x}dx+\int\frac{5}{2}dx$

$=-\ln|x|+\frac{5}{2}x+C_2$

$y(1)=2$ implies that $2=0+\frac{5}{2}+C_2$, so

$C_2=-\frac{1}{2}$. Thus

$y=-\ln|x|+\frac{5}{2}x-\frac{1}{2}=\ln\left|\frac{1}{x}\right|+\frac{5}{2}x-\frac{1}{2}$.

84. $y''=(x+1)^{3/2}$

$y'=\int(x+1)^{\frac{3}{2}}dx=\frac{2}{5}(x+1)^{\frac{5}{2}}+C_1$

$y'(3)=0\Rightarrow 0=\frac{2}{5}\cdot 32+C_1\Rightarrow C_1=-\frac{64}{5}$, so

$y'=\frac{2}{5}(x+1)^{\frac{5}{2}}-\frac{64}{5}$

$y=\int\left[\frac{2}{5}(x+1)^{\frac{5}{2}}-\frac{64}{5}\right]dx$

$=\frac{2}{5}\cdot\frac{(x+1)^{\frac{7}{2}}}{\frac{7}{2}}-\frac{64}{5}x+C_2$

$=\frac{4}{35}(x+1)^{\frac{7}{2}}-\frac{64}{5}x+C_2$

$y(3)=0$ implies $0=\frac{4}{35}\cdot 128-\frac{64}{5}(3)+C_2$, so

$C_2=\frac{832}{35}$. Thus $y=\frac{4}{35}(x+1)^{\frac{7}{2}}-\frac{64}{5}x+\frac{832}{35}$.

85. $V(t)=\int\frac{dV}{dt}dt=\int 8e^{0.05t}dt$

$=\frac{8}{0.05}\int e^{0.05t}[0.05\,dt]$

$=160e^{0.05t}+C$

The house cost \$350,000 to build, so $V(0)=350$.

$350=160e^0+C=160+C$

$190=C$

$V(t)=160e^{0.05t}+190$

86. $I(t)=\int\frac{dI}{dt}dt=\int\frac{12}{2t+50}dt$

$=6\ln|2t+50|+C$

Since the expected life span was 63 years in 1940, $I(0)=63$.

$63=6\ln|50|+C$

$C=63-6\ln 50\approx 39.53$

$I(t)=6\ln|2t+50|+39.53$

$I(58)=6\ln|166|+39.53\approx 70.20$

The expected life span for people born in 1998 (58 years after 1940) is about 70 years.

87. Note that $r>0$.

$C=\int\left[\frac{Rr}{2K}+\frac{B_1}{r}\right]dr=\int\frac{Rr}{2K}dr+\int\frac{B_1}{r}dr$

$=\frac{R}{2K}\int r\,dr+B_1\int\frac{1}{r}dr$

$=\frac{R}{2K}\cdot\frac{r^2}{2}+B_1\ln|r|+B_2$

Thus we obtain $C=\frac{Rr^2}{4K}+B_1\ln|r|+B_2$.

88. $f(x)=\int(e^{3x+2}-3x)dx=\frac{1}{3}e^{3x+2}-\frac{3}{2}x^2+C$

$f\left(\frac{1}{3}\right)=2$ implies $2=\frac{1}{3}e^3-\frac{1}{6}+C$, so

$C=\frac{13}{6}-\frac{1}{3}e^3$. Thus,

$f(x)=\frac{1}{3}e^{3x+2}-\frac{3}{2}x^2+\frac{13}{6}-\frac{1}{3}e^3$,

$f(2)=\frac{1}{3}e^8-6+\frac{13}{6}-\frac{1}{3}e^3$

$=\frac{1}{6}(2e^8-2e^3-23)\approx 983.12$

Problems 14.5

1. $\int\frac{2x^6+8x^4-4x}{2x^2}dx$

$=\int\left(\frac{2x^6}{2x^2}+\frac{8x^4}{2x^2}-\frac{4x}{2x^2}\right)dx$

$=\int x^4dx+4\int x^2dx-2\int\frac{1}{x}dx$

$=\frac{x^5}{5}+\frac{4}{3}x^3-2\ln|x|+C$

2. $\int\frac{9x^2+5}{3x}dx=\int\left(3x+\frac{5}{3x}\right)dx$

$=\frac{3}{2}x^2+\frac{5}{3}\ln|x|+C$

3. $\int\left(3x^2+2\right)\sqrt{2x^3+4x+1}\,dx$

$=\frac{1}{2}\int\left(2x^3+4x+1\right)^{\frac{1}{2}}\left[\left(6x^2+4\right)dx\right]$

$=\frac{1}{2}\cdot\frac{\left(2x^3+4x+1\right)^{\frac{3}{2}}}{\frac{3}{2}}+C$

$=\frac{1}{3}\left(2x^3+4x+1\right)^{\frac{3}{2}}+C$

4. $\int\frac{x}{\sqrt[4]{x^2+1}}dx=\frac{1}{2}\int\left(x^2+1\right)^{-\frac{1}{4}}[2x\,dx]$

$=\frac{1}{2}\cdot\frac{\left(x^2+1\right)^{\frac{3}{4}}}{\frac{3}{4}}+C$

$=\frac{2}{3}\left(x^2+1\right)^{\frac{3}{4}}+C$

5. $\int\frac{9}{\sqrt{2-3x}}dx=9\int(2-3x)^{-1/2}dx$

$=9\left(-\frac{1}{3}\right)\int(2-3x)^{-1/2}[-3\,dx]$

$=-3\frac{(2-3x)^{1/2}}{\frac{1}{2}}+C=-6\sqrt{2-3x}+C$

6. $\int\frac{2xe^{x^2}}{e^{x^2}-2}dx=\int\frac{1}{e^{x^2}-2}\left[2xe^{x^2}dx\right]$

$=\ln\left|e^{x^2}-2\right|+C$