18. $\frac{dy}{dx} = -1.5 - x$

$y = \int (-1.5 - x)dx = -1.5x - \frac{x^2}{2} + C$

When $x = 1$, then $y = 57.3$, so

$57.3 = -1.5 - 0.5 + C$, or $C = 59.3$. Thus $y = -1.5x - 0.5x^2 + 59.3$.

19. $v = \int -\frac{(P_1 - P_2)r}{2l\eta}dr = -\frac{(P_1 - P_2)r^2}{4l\eta} + C$

Since $v = 0$ when $r = R$, then $0 = -\frac{(P_1 - P_2)R^2}{4l\eta} + C$, so $C = \frac{(P_1 - P_2)R^2}{4l\eta}$. Thus

$v = -\frac{(P_1 - P_2)r^2}{4l\eta} + \frac{(P_1 - P_2)R^2}{4l\eta} = \frac{(P_1 - P_2)\left(R^2 - r^2\right)}{4l\eta}$.

20. $\frac{dr}{dq} = 100 - 3q^2$

$r = \int \left(100 - 3q^2\right)dq = 100q - q^3 + C$

When $q = 0$, then $r = 0$, so $C = 0$ and $r = 100q - q^3$. Since $r = pq$, then $p = \frac{r}{q} = 100 - q^2$.

$\eta = \frac{\frac{p}{q}}{\frac{dp}{dq}} = \frac{\frac{p}{q}}{-2q} = -\frac{p}{2q^2}$

When $q = 5$, then $p = 75$, so $\eta = \frac{-75}{2(25)} = -\frac{3}{2}$.

21. $\frac{dc}{dq} = 0.003q^2 - 0.4q + 40$

$c = \int \left(0.003q^2 - 0.4q + 40\right)dq = 0.001q^3 - 0.2q^2 + 40q + C$

When $q = 0$, then $c = 5000$, so

$5000 = 0 - 0 + 0 + C$, or $C = 5000$. Thus $c = 0.001q^3 - 0.2q^2 + 40q + 5000$. When $q = 100$, then $c = 8000$. Since Avg. Cost $= \bar{c} = \frac{\text{Total Cost}}{\text{Quantity}} = \frac{c}{q}$, when $q = 100$, we have $\bar{c} = \frac{8000}{100} = \$80$. (Observe that knowing $\frac{dc}{dq} = 27.50$ when $q = 50$ is not relevant to the problem.)

22. $f''(x) = 30x^4 + 12x$

$f'(x) = \int (30x^4 + 12x)dx = 6x^5 + 6x^2 + C_1$

$f'(1) = 10$, so $10 = 6 + 6 + C_1$ and $C_1 = -2$.

$f'(x) = 6x^5 + 6x^2 - 2$

$f(x) = \int (6x^5 + 6x^2 - 2)dx = x^6 + 2x^3 - 2x + C_2$

Thus

$f(965.335245) - f(-965.335245)$

$= [(965.335245)^6 + 2(965.335245)^3 - 2(965.335245) + C_2]$

$\quad - [(-965.335245)^6 + 2(-965.335245)^3 - 2(-965.335245) + C_2]$

$= 3{,}598{,}280{,}000$

Principles in Practice 14.4

1. Using the values given, $\frac{dT}{dt} = -0.5(70 - 60)e^{-0.5t} = -5e^{-0.5t}$

$T(t) = \int \frac{dT}{dt}dt = \int -5e^{-0.5t}dt = 10e^{-0.5t} + C$

2. The number of words memorized is $v(t)$.

$v(t) = \int \frac{35}{t+1}dt = 35\ln|t+1| + C$.

Problems 14.4

1. Let $u = x + 5 \Rightarrow du = 1dx = dx$

$\int (x+5)^7[dx] = \int u^7 du = \frac{u^8}{8} + C = \frac{(x+5)^8}{8} + C$

2. $\int 15(x+2)^4 dx = 15\int (x+2)^4[dx] = 15 \cdot \frac{(x+2)^5}{5} + C = 3(x+2)^5 + C$

3. Let $u = x^2 + 3 \Rightarrow du = 2x\,dx$

$\int 2x\left(x^2+3\right)^5 dx = \int \left(x^2+3\right)^5[2x\,dx] = \int u^5 du = \frac{u^6}{6} + C$

$= \frac{\left(x^2+3\right)^6}{6} + C$

4. Let $u = x^3 + 5x^2 + 6 \Rightarrow du = (3x^2 + 10x)dx$.

$\int \left(3x^2 + 10x\right)\left(x^3 + 5x^2 + 6\right)dx$

$= \int \left(x^3 + 5x^2 + 6\right)^1\left[\left(3x^2 + 10x\right)dx\right]$

$= \int u\,du = \frac{u^2}{2} + C$

$= \frac{\left(x^3 + 5x^2 + 6\right)^2}{2} + C$