Problems 14.3

1. $\frac{dy}{dx} = 3x - 4$

$y = \int (3x-4)dx = \frac{3x^2}{2} - 4x + C$

Using $y(-1) = \frac{13}{2}$ gives

$\frac{13}{2} = \frac{3(-1)^2}{2} - 4(-1) + C$

$\frac{13}{2} = \frac{11}{2} + C$

Thus $C = 1$, so $y = \frac{3x^2}{2} - 4x + 1$.

2. $\frac{dy}{dx} = x^2 - x$

$y = \int \left(x^2 - x\right)dx = \frac{x^3}{3} - \frac{x^2}{2} + C$

Using $y(3) = \frac{19}{2}$ gives $\frac{19}{2} = \frac{3^3}{3} - \frac{3^2}{2} + C$

$\frac{19}{2} = \frac{9}{2} + C$

Thus, $C = 5$, so $y = \frac{x^3}{3} - \frac{x^2}{2} + 5$.

3. $y' = \frac{5}{\sqrt{x}}$

$y = \int \frac{5}{\sqrt{x}}dx = \int 5x^{-\frac{1}{2}}dx = 5 \cdot \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C = 10\sqrt{x} + C$

$y(9) = 50$ implies $50 = 10\sqrt{9} + C$, $50 = 30 + C$, $C = 20$.

Thus $y = 10\sqrt{x} + 20$.

$y(16) = 10 \cdot 4 + 20 = 60$

4. $y' = -x^2 + 2x$

$y = \int \left(-x^2 + 2x\right)dx = -\frac{x^3}{3} + x^2 + C$

$y(2) = 1$ implies $1 = -\frac{8}{3} + 4 + C$, so $C = -\frac{1}{3}$.

Thus $y = -\frac{x^3}{3} + x^2 - \frac{1}{3}$.

$y(1) = -\frac{1}{3} + 1 - \frac{1}{3} = \frac{1}{3}$

5. $y'' = -3x^2 + 4x$

$y' = \int (-3x^2 + 4x)dx = -x^3 + 2x^2 + C_1$

$y'(1) = 2$ implies $2 = -1 + 2 + C_1$, so $C_1 = 1$.

$y = \int (-x^3 + 2x^2 + 1)dx = -\frac{x^4}{4} + \frac{2x^3}{3} + x + C_2$

$y(1) = 3$ implies $3 = -\frac{1}{4} + \frac{2}{3} + 1 + C_2$, so

$C_2 = \frac{19}{12}$. Thus $y = -\frac{x^4}{4} + \frac{2x^3}{3} + x + \frac{19}{12}$.

6. $y'' = x + 1$

$y' = \int (x+1)dx = \frac{x^2}{2} + x + C_1$

$y'(0) = 0$ implies $0 = 0 + 0 + C_1$, so $C_1 = 0$.

$y = \int \left[\frac{x^2}{2} + x\right]dx = \frac{x^3}{6} + \frac{x^2}{2} + C_2$.

$y(0) = 5$ implies $5 = 0 + 0 + C_2$, so $C_2 = 5$. Thus

$y = \frac{x^3}{6} + \frac{x^2}{2} + 5$.

7. $y''' = 2x$

$y'' = \int 2x\, dx = x^2 + C_1$

$y''(-1) = 3$ implies that $3 = 1 + C_1$, so $C_1 = 2$.

$y' = \int \left(x^2 + 2\right)dx = \frac{x^3}{3} + 2x + C_2$

$y'(3) = 10$ implies $10 = 9 + 6 + C_2$, so $C_2 = -5$.

$y = \int \left(\frac{x^3}{3} + 2x - 5\right)dx = \frac{x^4}{12} + x^2 - 5x + C_3$.

$y(0) = 13$ implies that $13 = 0 + 0 - 0 + C_3$, so

$C_3 = 13$. Therefore $y = \frac{x^4}{12} + x^2 - 5x + 13$.

8. $y''' = e^x + 1$

$y'' = \int \left(e^x + 1\right)dx = e^x + x + C_1$

$y''(0) = 1$ implies $1 = 1 + 0 + C_1$, so $C_1 = 0$.

$y' = \int \left(e^x + x\right)dx = e^x + \frac{x^2}{2} + C_2$

$y'(0) = 2$ implies $2 = 1 + 0 + C_2$, so $C_2 = 1$.

$y = \int \left[e^x + \frac{x^2}{2} + 1\right]dx = e^x + \frac{x^3}{6} + x + C_3$

$y(0) = 3$ implies that $3 = 1 + 0 + 0 + C_3$, so

$C_3 = 2$. Thus $y = e^x + \frac{x^3}{6} + x + 2$.

9. $\frac{dr}{dq} = 0.7$

$r = \int 0.7dq = 0.7q + C$

If $q = 0$, r must be 0, so $0 = 0 + C$, $C = 0$. Thus $r = 0.7q$. Since $r = pq$, we have

$p = \frac{r}{q} = \frac{0.7q}{q} = 0.7$. The demand function is

$p = 0.7$.

10. $\frac{dr}{dq} = 10 - \frac{1}{16}q$

$r = \int \left[10 - \frac{1}{16}q\right]dq = 10q - \frac{1}{32}q^2 + C$

When $q = 0$, then $r = 0$, so $C = 0$ and

$r = 10q - \frac{1}{32}q^2$. Since $r = pq$, then

$p = \frac{r}{q} = 10 - \frac{1}{32}q$. The demand function is

$p = 10 - \frac{1}{32}q$.

11. $\frac{dr}{dq} = 275 - q - 0.3q^2$

Thus $r = \int \left(275 - q - 0.3q^2\right)dq$

$= 275q - 0.5q^2 - 0.1q^3 + C$. When $q = 0$, r must be 0, so $C = 0$ and $r = 275q - 0.5q^2 - 0.1q^3$.

Since $r = pq$, then $p = \frac{r}{q} = 275 - 0.5q - 0.1q^2$.

Thus the demand function is

$p = 275 - 0.5q - 0.1q^2$.

12. $\frac{dr}{dq} = 5000 - 3(2q + 2q^3)$, so

$r = \int (5000 - 6q - 6q^3)dq$

$= 5000q - 3q^2 - \frac{3q^4}{2} + C$

When $q = 0$, then $r = 0$, so $C = 0$ and

$r = 5000q - 3q^2 - \frac{3q^4}{2}$. Since $r = pq$, then

$p = \frac{r}{q} = 5000 - 3q - \frac{3q^3}{2}$. Therefore the demand

function is $p = 5000 - 3q - \frac{3q^3}{2}$.

13. $\frac{dc}{dq} = 1.35$

$c = \int 1.35dq = 1.35q + C$

When $q = 0$, then $c = 200$, so $200 = 0 + C$, or $C = 200$. Thus $c = 1.35q + 200$.

14. $\frac{dc}{dq} = 2q + 75$

$c = \int (2q + 75)dq = q^2 + 75q + C$

When $q = 0$, then $c = 2000$, so $C = 2000$. Thus the cost function is $c = q^2 + 75q + 2000$.

15. $\frac{dc}{dq} = 0.08q^2 - 1.6q + 6.5$

$c = \int \left(0.08q^2 - 1.6q + 6.5\right)dq$

$\frac{0.08}{3}q^3 - 0.8q^2 + 6.5q + C$. If $q = 0$, then

$c = 8000$, from which $C = 8000$. Hence

$c = \frac{0.08}{3}q^3 - 0.8q^2 + 6.5q + 8000$. If $q = 25$,

substituting gives $c(25) = 8079\frac{1}{6}$ or \$8079.17.

16. $\frac{dc}{dq} = 0.000204q^2 - 0.046q + 6$

$c = \int (0.000204q^2 - 0.046q + 6)dq$

$= 0.000068q^3 - 0.023q^2 + 6q + C$

When $q = 0$, then $c = 15{,}000$, from which $C = 15{,}000$. The cost function is

$c = 0.000068q^3 - 0.023q^2 + 6q + 15{,}000$. When $q = 200$, substitution gives $c(200) = 15{,}824$.

17. $G = \int \left[-\frac{P}{25} + 2\right]dP = -\frac{P^2}{50} + 2P + C$

When $P = 10$, then $G = 38$, so $38 = -2 + 20 + C$, from which $C = 20$. Thus

$G = -\frac{1}{50}P^2 + 2P + 20$.