Problems 14.3

1. $\frac{dy}{dx} = 3x - 4$

   $y = \int (3x-4)dx = \frac{3x^2}{2} - 4x + C$

   Using $y(-1) = \frac{13}{2}$ gives

   $\frac{13}{2} = \frac{3(-1)^2}{2} - 4(-1) + C$

   $\frac{13}{2} = \frac{11}{2} + C$

   Thus $C = 1$, so $y = \frac{3x^2}{2} - 4x + 1$.

2. $\frac{dy}{dx} = x^2 - x$

   $y = \int \left(x^2 - x\right)dx = \frac{x^3}{3} - \frac{x^2}{2} + C$

   Using $y(3) = \frac{19}{2}$ gives $\frac{19}{2} = \frac{3^3}{3} - \frac{3^2}{2} + C$

   $\frac{19}{2} = \frac{9}{2} + C$

   Thus, $C = 5$, so $y = \frac{x^3}{3} - \frac{x^2}{2} + 5$.

3. $y' = \frac{5}{\sqrt{x}}$

   $y = \int \frac{5}{\sqrt{x}}dx = \int 5x^{-\frac{1}{2}}dx = 5 \cdot \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C = 10\sqrt{x} + C$

   $y(9) = 50$ implies $50 = 10\sqrt{9} + C$, $50 = 30 + C$, $C = 20$.

   Thus $y = 10\sqrt{x} + 20$.

   $y(16) = 10 \cdot 4 + 20 = 60$

4. $y' = -x^2 + 2x$

   $y = \int \left(-x^2 + 2x\right)dx = -\frac{x^3}{3} + x^2 + C$

   $y(2) = 1$ implies $1 = -\frac{8}{3} + 4 + C$, so $C = -\frac{1}{3}$.

   Thus $y = -\frac{x^3}{3} + x^2 - \frac{1}{3}$.

   $y(1) = -\frac{1}{3} + 1 - \frac{1}{3} = \frac{1}{3}$

5. $y'' = -3x^2 + 4x$

   $y' = \int (-3x^2 + 4x)dx = -x^3 + 2x^2 + C_1$

   $y'(1) = 2$ implies $2 = -1 + 2 + C_1$, so $C_1 = 1$.

   $y = \int (-x^3 + 2x^2 + 1)dx = -\frac{x^4}{4} + \frac{2x^3}{3} + x + C_2$

   $y(1) = 3$ implies $3 = -\frac{1}{4} + \frac{2}{3} + 1 + C_2$, so

   $C_2 = \frac{19}{12}$. Thus $y = -\frac{x^4}{4} + \frac{2x^3}{3} + x + \frac{19}{12}$.

6. $y'' = x + 1$

   $y' = \int (x+1)dx = \frac{x^2}{2} + x + C_1$

   $y'(0) = 0$ implies $0 = 0 + 0 + C_1$, so $C_1 = 0$.

   $y = \int \left[\frac{x^2}{2} + x\right]dx = \frac{x^3}{6} + \frac{x^2}{2} + C_2$.

   $y(0) = 5$ implies $5 = 0 + 0 + C_2$, so $C_2 = 5$. Thus

   $y = \frac{x^3}{6} + \frac{x^2}{2} + 5$.

7. $y''' = 2x$

   $y'' = \int 2x\ dx = x^2 + C_1$

   $y''(-1) = 3$ implies that $3 = 1 + C_1$, so $C_1 = 2$.

   $y' = \int \left(x^2 + 2\right)dx = \frac{x^3}{3} + 2x + C_2$

   $y'(3) = 10$ implies $10 = 9 + 6 + C_2$, so $C_2 = -5$.

   $y = \int \left(\frac{x^3}{3} + 2x - 5\right)dx = \frac{x^4}{12} + x^2 - 5x + C_3$.

   $y(0) = 13$ implies that $13 = 0 + 0 - 0 + C_3$, so

   $C_3 = 13$. Therefore $y = \frac{x^4}{12} + x^2 - 5x + 13$.

8. $y''' = e^x + 1$

   $y'' = \int \left(e^x + 1\right)dx = e^x + x + C_1$

   $y''(0) = 1$ implies $1 = 1 + 0 + C_1$, so $C_1 = 0$.

   $y' = \int \left(e^x + x\right)dx = e^x + \frac{x^2}{2} + C_2$

   $y'(0) = 2$ implies $2 = 1 + 0 + C_2$, so $C_2 = 1$.

   $y = \int \left[e^x + \frac{x^2}{2} + 1\right]dx = e^x + \frac{x^3}{6} + x + C_3$

   $y(0) = 3$ implies that $3 = 1 + 0 + 0 + C_3$, so

   $C_3 = 2$. Thus $y = e^x + \frac{x^3}{6} + x + 2$.

9. $\frac{dr}{dq} = 0.7$

   $r = \int 0.7dq = 0.7q + C$

   If $q = 0$, r must be 0, so $0 = 0 + C$, $C = 0$. Thus $r = 0.7q$. Since $r = pq$, we have

   $p = \frac{r}{q} = \frac{0.7q}{q} = 0.7$. The demand function is $p = 0.7$.

10. $\frac{dr}{dq} = 10 - \frac{1}{16}q$

    $r = \int \left[10 - \frac{1}{16}q\right]dq = 10q - \frac{1}{32}q^2 + C$

    When $q = 0$, then $r = 0$, so $C = 0$ and

    $r = 10q - \frac{1}{32}q^2$. Since $r = pq$, then

    $p = \frac{r}{q} = 10 - \frac{1}{32}q$. The demand function is

    $p = 10 - \frac{1}{32}q$.

11. $\frac{dr}{dq} = 275 - q - 0.3q^2$

    Thus $r = \int \left(275 - q - 0.3q^2\right)dq$

    $= 275q - 0.5q^2 - 0.1q^3 + C$. When $q = 0$, r must be 0, so $C = 0$ and $r = 275q - 0.5q^2 - 0.1q^3$.

    Since $r = pq$, then $p = \frac{r}{q} = 275 - 0.5q - 0.1q^2$.

    Thus the demand function is

    $p = 275 - 0.5q - 0.1q^2$.

12. $\frac{dr}{dq} = 5000 - 3(2q + 2q^3)$, so

    $r = \int (5000 - 6q - 6q^3)dq$

    $= 5000q - 3q^2 - \frac{3q^4}{2} + C$

    When $q = 0$, then $r = 0$, so $C = 0$ and

    $r = 5000q - 3q^2 - \frac{3q^4}{2}$. Since $r = pq$, then

    $p = \frac{r}{q} = 5000 - 3q - \frac{3q^3}{2}$. Therefore the demand function is $p = 5000 - 3q - \frac{3q^3}{2}$.

13. $\frac{dc}{dq} = 1.35$

    $c = \int 1.35dq = 1.35q + C$

    When $q = 0$, then $c = 200$, so $200 = 0 + C$, or $C = 200$. Thus $c = 1.35q + 200$.

14. $\frac{dc}{dq} = 2q + 75$

    $c = \int (2q + 75)dq = q^2 + 75q + C$

    When $q = 0$, then $c = 2000$, so $C = 2000$. Thus the cost function is $c = q^2 + 75q + 2000$.

15. $\frac{dc}{dq} = 0.08q^2 - 1.6q + 6.5$

    $c = \int \left(0.08q^2 - 1.6q + 6.5\right)dq$

    $\frac{0.08}{3}q^3 - 0.8q^2 + 6.5q + C$. If $q = 0$, then $c = 8000$, from which $C = 8000$. Hence

    $c = \frac{0.08}{3}q^3 - 0.8q^2 + 6.5q + 8000$. If $q = 25$,

    substituting gives $c(25) = 8079\frac{1}{6}$ or \$8079.17.

16. $\frac{dc}{dq} = 0.000204q^2 - 0.046q + 6$

    $c = \int (0.000204q^2 - 0.046q + 6)dq$

    $= 0.000068q^3 - 0.023q^2 + 6q + C$

    When $q = 0$, then $c = 15,000$, from which $C = 15,000$. The cost function is

    $c = 0.000068q^3 - 0.023q^2 + 6q + 15,000$. When $q = 200$, substitution gives $c(200) = 15,824$.

17. $G = \int \left[-\frac{P}{25} + 2\right]dP = -\frac{P^2}{50} + 2P + C$

    When $P = 10$, then $G = 38$, so $38 = -2 + 20 + C$, from which $C = 20$. Thus

    $G = -\frac{1}{50}P^2 + 2P + 20$.

18. $\frac{dy}{dx} = -1.5 - x$

$y = \int(-1.5 - x)dx = -1.5x - \frac{x^2}{2} + C$

When $x = 1$, then $y = 57.3$, so

$57.3 = -1.5 - 0.5 + C$, or $C = 59.3$. Thus $y = -1.5x - 0.5x^2 + 59.3$.

19. $v = \int -\frac{(P_1 - P_2)r}{2l\eta}dr = -\frac{(P_1 - P_2)r^2}{4l\eta} + C$

Since $v = 0$ when $r = R$, then $0 = -\frac{(P_1 - P_2)R^2}{4l\eta} + C$, so $C = \frac{(P_1 - P_2)R^2}{4l\eta}$. Thus

$v = -\frac{(P_1 - P_2)r^2}{4l\eta} + \frac{(P_1 - P_2)R^2}{4l\eta} = \frac{(P_1 - P_2)\left(R^2 - r^2\right)}{4l\eta}$.

20. $\frac{dr}{dq} = 100 - 3q^2$

$r = \int\left(100 - 3q^2\right)dq = 100q - q^3 + C$

When $q = 0$, then $r = 0$, so $C = 0$ and $r = 100q - q^3$. Since $r = pq$, then $p = \frac{r}{q} = 100 - q^2$.

$\eta = \frac{\frac{p}{q}}{\frac{dp}{dq}} = \frac{\frac{p}{q}}{-2q} = -\frac{p}{2q^2}$

When $q = 5$, then $p = 75$, so $\eta = \frac{-75}{2(25)} = -\frac{3}{2}$.

21. $\frac{dc}{dq} = 0.003q^2 - 0.4q + 40$

$c = \int\left(0.003q^2 - 0.4q + 40\right)dq = 0.001q^3 - 0.2q^2 + 40q + C$

When $q = 0$, then $c = 5000$, so

$5000 = 0 - 0 + 0 + C$, or $C = 5000$. Thus $c = 0.001q^3 - 0.2q^2 + 40q + 5000$. When $q = 100$, then $c = 8000$. Since Avg. Cost $= \overline{c} = \frac{\text{Total Cost}}{\text{Quantity}} = \frac{c}{q}$, when $q = 100$, we have $\overline{c} = \frac{8000}{100} = \$80$. (Observe that knowing $\frac{dc}{dq} = 27.50$ when $q = 50$ is not relevant to the problem.)

22. $f''(x) = 30x^4 + 12x$

$f'(x) = \int(30x^4 + 12x)dx = 6x^5 + 6x^2 + C_1$

$f'(1) = 10$, so $10 = 6 + 6 + C_1$ and $C_1 = -2$.

$f'(x) = 6x^5 + 6x^2 - 2$

$f(x) = \int(6x^5 + 6x^2 - 2)dx = x^6 + 2x^3 - 2x + C_2$

Thus

$f(965.335245) - f(-965.335245)$

$= [(965.335245)^6 + 2(965.335245)^3 - 2(965.335245) + C_2]$
$\quad - [(-965.335245)^6 + 2(-965.335245)^3 - 2(-965.335245) + C_2]$

$= 3,598,280,000$

Principles in Practice 14.4

1. Using the values given, $\frac{dT}{dt} = -0.5(70 - 60)e^{-0.5t} = -5e^{-0.5t}$

$T(t) = \int\frac{dT}{dt}dt = \int -5e^{-0.5t}dt = 10e^{-0.5t} + C$

2. The number of words memorized is $v(t)$.

$v(t) = \int\frac{35}{t+1}dt = 35\ln|t+1| + C$.

Problems 14.4

1. Let $u = x + 5 \Rightarrow du = 1dx = dx$

$\int(x+5)^7[dx] = \int u^7 du = \frac{u^8}{8} + C = \frac{(x+5)^8}{8} + C$

2. $\int 15(x+2)^4 dx = 15\int(x+2)^4[dx] = 15 \cdot \frac{(x+2)^5}{5} + C = 3(x+2)^5 + C$

3. Let $u = x^2 + 3 \Rightarrow du = 2x\ dx$

$\int 2x\left(x^2+3\right)^5 dx = \int\left(x^2+3\right)^5[2x\ dx] = \int u^5 du = \frac{u^6}{6} + C$

$= \frac{\left(x^2+3\right)^6}{6} + C$

4. Let $u = x^3 + 5x^2 + 6 \Rightarrow du = (3x^2 + 10x)dx$.

$\int\left(3x^2 + 10x\right)\left(x^3 + 5x^2 + 6\right)dx$

$= \int\left(x^3 + 5x^2 + 6\right)^1\left[\left(3x^2 + 10x\right)dx\right]$

$= \int u\ du = \frac{u^2}{2} + C$

$= \frac{\left(x^3 + 5x^2 + 6\right)^2}{2} + C$