39. $\int\left(-\frac{\sqrt[3]{x^2}}{5}-\frac{7}{2\sqrt{x}}+6x\right)dx$

$=\int\left(-\frac{x^{\frac{2}{3}}}{5}-\frac{7x^{-\frac{1}{2}}}{2}+6x\right)dx$

$=-\frac{1}{5}\int x^{\frac{2}{3}}dx-\frac{7}{2}\int x^{-\frac{1}{2}}dx+6\int x\,dx$

$=-\frac{1}{5}\cdot\frac{x^{\frac{5}{3}}}{\frac{5}{3}}-\frac{7}{2}\cdot\frac{x^{\frac{1}{2}}}{\frac{1}{2}}+6\cdot\frac{x^2}{2}+C$

$=-\frac{3x^{\frac{5}{3}}}{25}-7x^{\frac{1}{2}}+3x^2+C$

40. $\int\left(\sqrt[3]{u}+\frac{1}{\sqrt{u}}\right)du=\int\left(u^{\frac{1}{3}}+u^{-\frac{1}{2}}\right)du$

$=\int u^{\frac{1}{3}}du+\int u^{-\frac{1}{2}}du$

$=\frac{u^{\frac{4}{3}}}{\frac{4}{3}}+\frac{u^{\frac{1}{2}}}{\frac{1}{2}}+C$

$=\frac{3u^{\frac{4}{3}}}{4}+2u^{\frac{1}{2}}+C$

41. $\int(x^2+5)(x-3)dx=\int(x^3-3x^2+5x-15)dx$

$=\frac{x^4}{4}-3\cdot\frac{x^3}{3}+5\cdot\frac{x^2}{2}-15x+C$

$=\frac{x^4}{4}-x^3+\frac{5x^2}{2}-15x+C$

42. $\int x^4(x^3+8x^2+7)dx=\int(x^7+8x^6+7x^4)dx$

$=\frac{x^8}{8}+8\cdot\frac{x^7}{7}+7\cdot\frac{x^5}{5}+C$

$=\frac{x^8}{8}+\frac{8x^7}{7}+\frac{7x^5}{5}+C$

43. $\int\sqrt{x}(x+3)dx=\int\left(x^{\frac{3}{2}}+3x^{\frac{1}{2}}\right)dx$

$=\frac{x^{\frac{5}{2}}}{\frac{5}{2}}+3\cdot\frac{x^{\frac{3}{2}}}{\frac{3}{2}}+C$

$=\frac{2x^{\frac{5}{2}}}{5}+2x^{\frac{3}{2}}+C$

44. $\int(z+2)^2dz=\int(z^2+4z+4)dz$

$=\frac{z^3}{3}+4\cdot\frac{z^2}{2}+4z+C$

$=\frac{z^3}{3}+2z^2+4z+C$

45. $\int(3u+2)^3du=\int(27u^3+54u^2+36u+8)du$

$=27\cdot\frac{u^4}{4}+54\cdot\frac{u^3}{3}+36\cdot\frac{u^2}{2}+8u+C$

$=\frac{27}{4}u^4+18u^3+18u^2+8u+C$

46. $\int\left(\frac{2}{\sqrt[5]{x}}-1\right)^2dx=\int\left(2x^{-\frac{1}{5}}-1\right)^2dx$

$=\int\left(4x^{-\frac{2}{5}}-4x^{-\frac{1}{5}}+1\right)dx$

$=4\cdot\frac{x^{\frac{3}{5}}}{\frac{3}{5}}-4\cdot\frac{x^{\frac{4}{5}}}{\frac{4}{5}}+x+C$

$=\frac{20x^{\frac{3}{5}}}{3}-5x^{\frac{4}{5}}+x+C$

47. $\int v^{-2}\left(2v^4+3v^2-2v^{-3}\right)dv$

$=\int\left(2v^2+3-2v^{-5}\right)dv$

$=2\cdot\frac{v^3}{3}+3v-2\cdot\frac{v^{-4}}{-4}+C$

$=\frac{2v^3}{3}+3v+\frac{1}{2v^4}+C$

48. $\int\left[6e^u-u^3\left(\sqrt{u}+1\right)\right]du=\int\left[6e^u-u^{\frac{7}{2}}-u^3\right]du$

$=6\cdot e^u-\frac{u^{\frac{9}{2}}}{\frac{9}{2}}-\frac{u^4}{4}+C$

$=6e^u-\frac{2u^{\frac{9}{2}}}{9}-\frac{u^4}{4}+C$

49. $\int\frac{z^4+10z^3}{2z^2}dz=\frac{1}{2}\int\left(\frac{z^4}{z^2}+\frac{10z^3}{z^2}\right)dz$

$=\frac{1}{2}\int\left(z^2+10z\right)dz$

$=\frac{1}{2}\left(\frac{z^3}{3}+10\cdot\frac{z^2}{2}\right)+C$

$=\frac{z^3}{6}+\frac{5z^2}{2}+C$

50. $\int\frac{x^4-5x^2+2x}{5x^2}dx=\frac{1}{5}\int\left(x^2-5+\frac{2}{x}\right)dx$

$=\frac{1}{5}\left(\frac{x^3}{3}-5x+2\ln|x|\right)+C$

51. $\int\frac{e^x+e^{2x}}{e^x}dx=\int\left(\frac{e^x}{e^x}+\frac{e^{2x}}{e^x}\right)dx$

$=\int\left(1+e^x\right)dx$

$=x+e^x+C$

52. $\int\frac{(x^3+1)^2}{x^2}dx=\int\frac{x^6+2x^3+1}{x^2}dx$

$=\int(x^4+2x+x^{-2})dx$

$=\frac{x^5}{5}+2\cdot\frac{x^2}{2}+\frac{x^{-1}}{-1}+C$

$=\frac{x^5}{5}+x^2-\frac{1}{x}+C$

53. No, $F(x)-G(x)$ might be a nonzero constant.

54. **a.** $F(x)=\frac{d}{dx}\left(xe^x\right)=xe^x+e^x(1)=e^x(x+1)$

b. There is only one.

55. Because an antiderivative of the derivative of a function is the function itself, we have

$\int\frac{d}{dx}\left[\frac{1}{\sqrt{x^2+1}}\right]dx=\frac{1}{\sqrt{x^2+1}}+C$.

Principles in Practice 14.3

1. $N(t)=\int\frac{dN}{dt}dt=\int\left(800+200e^t\right)dt$

$=800t+200e^t+C$

Since $N(5)=40{,}000$, we have

$40{,}000=800(5)+200e^5+C$, so

$C=40{,}000-\left(4000+200e^5\right)$

$=36{,}000-200e^5\approx 6317.37$

$N(t)=800t+200e^t+6317.37$

2. Since $y''=\frac{d}{dt}(y')=84t+24$

$y'=\int(84t+24)dt=84\left(\frac{t^2}{2}\right)+24t+C_1$

$=42t^2+24t+C_1$

Since $y'(8)=2891$, we have

$2891=42(8)^2+24(8)+C_1=2880+C_1$, so

$C_1=2891-2880=11$, and $y'=42t^2+24t+11$.

$y(t)=\int y'dt=\int\left(42t^2+24t+11\right)dt$

$=42\left(\frac{t^3}{3}\right)+24\left(\frac{t^2}{2}\right)+11t+C_2$

$=14t^3+12t^2+11t+C_2$

Since $y(2)=185$, we have

$185=14(2)^3+12(2)^2+11(2)+C_2$

$=182+C_2$, so $C_2=185-182=3$.

$y(t)=14t^3+12t^2+11t+3$