$\Delta V \approx 4\pi\left(6.5\times10^{-4}\right)^2\left(10^{-5}\right) = \left(1.69\times10^{-11}\right)\pi \text{ cm}^3$.

44. $(P + a)(v + b) = k$

$$P = \frac{k}{v+b} - a$$

$$dP = -k(v+b)^{-2}dv$$

45. a. We substitute $q = 40$ and $p = 20$

$$2 + \frac{40^2}{200} = \frac{4000}{20^2}$$
$$2 + 8 = 10$$
$$10 = 10$$

b. We differentiate implicitly with respect to p.

$$0 + \frac{1}{200}\left(2q\frac{dq}{dp}\right) = -\frac{8000}{p^3}$$

From part (a) $q = 40$ when $p = 20$. Substituting gives

$$\frac{1}{200}\left(2\cdot 40\frac{dq}{dp}\right) = -\frac{8000}{20^3}$$
$$\frac{dq}{dp} = -2.5$$

c. $q(p+dp) \approx q(p) + dq = q(p) + q'(p)dp$

$$q(19.20) = q(20 + (-0.8))$$
$$\approx q(20) + q'(20)dp$$
$$= 40 + (-2.5)(-0.8)$$
$$= 42 \text{ units}$$

46. a. Profit $= TR - TC = pq - \bar{c}q$

$$P = \frac{1}{2}q^3 - 66q^2 + 7000q - \left(500q - q^2 + \frac{80{,}000}{2}\right) = \frac{1}{2}q^3 - 65q^2 + 6500q - 40{,}000$$

If $q = 100$, then $P = \frac{1}{2}(100)^3 - 65(100)^2 + 6500(100) - 40{,}000 = 460{,}000$

b. We use $P(q + dq) \approx P(q) + dP$ with $q = 100$ and $dq = -2$.

$$P(98) = P(100 + (-2))$$
$$\approx P(100) + \left(\frac{3}{2}q^2 - 130q + 6500\right)dq$$
$$= 460{,}000 + \left[\frac{3}{2}(100)^2 - 130(100) + 6500\right](-2)$$
$$= \$443{,}000$$

Principles in Practice 14.2

1. $\int 28.3\,dq = 28.3q + C$

The form of the cost function is $28.3q + C$.

2. $\int 0.12t^2 dt = 0.12\frac{t^3}{3} + C = 0.04t^3 + C$

The form of the revenue function is $R(t) = 0.04t^3 + C$.

3. Let $S(t)$ = the number of subscribers t months after the competition entered the market, then $S'(t) = -\frac{480}{t^3}$.

$$S(t) = \int -\frac{480}{t^3}dt = -480\int t^{-3}dt$$
$$= -480\left(\frac{t^{-2}}{-2}\right) + C = 240t^{-2} + C = \frac{240}{t^2} + C$$

The number of subscribers is $S(t) = \frac{240}{t^2} + C$.

4. $\int\left(500 + 300\sqrt{t}\right)dt = \int\left(500 + 300t^{\frac{1}{2}}\right)dt$

$$= 500t + \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + C = 500t + \frac{2}{3}t^{\frac{3}{2}} + C$$

The population is $N(t) = 500t + \frac{2}{3}t^{\frac{3}{2}} + C$

5. The amount of money saved is $\int\frac{dS}{dt}dt$.

$$\int\left(2.1t^2 - 65.4t + 491.6\right)dt$$
$$= 2.1\left(\frac{t^3}{3}\right) - 65.4\left(\frac{t^2}{2}\right) + 491.6t + C$$
$$= 0.7t^3 - 32.7t^2 + 491.6t + C$$

The amount of money saved is $S(t) = 0.7t^3 - 32.7t^2 + 491.6t + C$

Problems 14.2

1. $\int 7\,dx = 7x + C$

2. $\int\frac{1}{2x}dx = \frac{1}{2}\int\frac{1}{x}dx = \frac{1}{2}\ln|x| + C$

3. $\int x^8 dx = \frac{x^{8+1}}{8+1} + C = \frac{x^9}{9} + C$

4. $\int 5x^{24}\,dx = 5\int x^{24}\,dx = 5\cdot\frac{x^{24+1}}{24+1} + C$

$$= 5\cdot\frac{x^{25}}{25} + C = \frac{x^{25}}{5} + C$$

5. $\int 5x^{-7}dx = 5\int x^{-7}dx = 5\cdot\frac{x^{-7+1}}{-7+1} + C$

$$= 5\cdot\frac{x^{-6}}{-6} + C = -\frac{5}{6x^6} + C$$

6. $\int\frac{z^{-3}}{3}dz = \frac{1}{3}\int z^{-3}dz = \frac{1}{3}\cdot\frac{z^{-3+1}}{-3+1} + C$

$$= \frac{1}{3}\cdot\frac{z^{-2}}{-2} + C = -\frac{1}{6z^2} + C$$

7. $\int\frac{2}{x^{10}}dx = 2\int x^{-10}dx = 2\cdot\frac{x^{-10+1}}{-10+1} + C$

$$= \frac{2x^{-9}}{-9} + C = -\frac{2}{9x^9} + C$$

8. $\int\frac{7}{x^4}dx = 7\int x^{-4}dx = 7\cdot\frac{x^{-4+1}}{-4+1} + C = \frac{7x^{-3}}{-3} + C$

$$= -\frac{7}{3x^3} + C$$

9. $\int\frac{1}{t^{7/4}}dt = \int t^{-7/4}dt = \frac{t^{-7/4+1}}{-\frac{7}{4}+1} + C = \frac{t^{-3/4}}{-\frac{3}{4}} + C$

$$= -\frac{4}{3t^{3/4}} + C$$

10. $\int\frac{7}{2x^{\frac{9}{4}}}dx = \frac{7}{2}\int x^{-\frac{9}{4}}dx = \frac{7}{2}\cdot\frac{x^{\frac{-9}{4}+1}}{\frac{-9}{4}+1} + C$

$$= \frac{7}{2}\cdot\frac{x^{-\frac{5}{4}}}{-\frac{5}{4}} + C$$
$$= -\frac{14}{5x^{\frac{5}{4}}} + C$$