$$\Delta V \approx 4\pi\left(6.5\times10^{-4}\right)^2\left(10^{-5}\right) = \left(1.69\times10^{-11}\right)\pi \text{ cm}^3.$$

44. $(P + a)(v + b) = k$

$$P = \frac{k}{v+b} - a$$

$$dP = -k(v+b)^{-2}\,dv$$

45. a. We substitute $q = 40$ and $p = 20$

$$2+\frac{40^2}{200} = \frac{4000}{20^2}$$
$$2+8=10$$
$$10=10$$

b. We differentiate implicitly with respect to p.

$$0+\frac{1}{200}\left(2q\frac{dq}{dp}\right) = -\frac{8000}{p^3}$$

From part (a) $q = 40$ when $p = 20$. Substituting gives

$$\frac{1}{200}\left(2\cdot 40\frac{dq}{dp}\right) = -\frac{8000}{20^3}$$
$$\frac{dq}{dp} = -2.5$$

c. $q(p+dp) \approx q(p)+dq = q(p)+q'(p)dp$

$$\begin{aligned} q(19.20) &= q(20+(-0.8)) \\ &\approx q(20)+q'(20)dp \\ &= 40+(-2.5)(-0.8) \\ &= 42 \text{ units} \end{aligned}$$

46. a. Profit $= TR - TC = pq - \bar{c}q$

$$P = \frac{1}{2}q^3 - 66q^2 + 7000q - \left(500q - q^2 + \frac{80,000}{2}\right) = \frac{1}{2}q^3 - 65q^2 + 6500q - 40,000$$

If $q = 100$, then $P = \frac{1}{2}(100)^3 - 65(100)^2 + 6500(100) - 40,000 = 460,000$

b. We use $P(q + dq) \approx P(q) + dP$ with $q = 100$ and $dq = -2$.

$$\begin{aligned} P(98) &= P(100+(-2)) \\ &\approx P(100) + \left(\frac{3}{2}q^2 - 130q + 6500\right)dq \\ &= 460,000 + \left[\frac{3}{2}(100)^2 - 130(100) + 6500\right](-2) \\ &= \$443,000 \end{aligned}$$

Principles in Practice 14.2

1. $\int 28.3\,dq = 28.3q + C$

The form of the cost function is $28.3q + C$.

2. $\int 0.12t^2 dt = 0.12\frac{t^3}{3} + C = 0.04t^3 + C$

The form of the revenue function is $R(t) = 0.04t^3 + C$.

3. Let $S(t)$ = the number of subscribers t months after the competition entered the market, then $S'(t) = -\frac{480}{t^3}$.

$$\begin{aligned} S(t) &= \int -\frac{480}{t^3}dt = -480\int t^{-3}dt \\ &= -480\left(\frac{t^{-2}}{-2}\right) + C = 240t^{-2} + C = \frac{240}{t^2} + C \end{aligned}$$

The number of subscribers is $S(t) = \frac{240}{t^2} + C$.

4. $\int\left(500+300\sqrt{t}\right)dt = \int\left(500+300t^{\frac{1}{2}}\right)dt$

$$= 500t + \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + C = 500t + \frac{2}{3}t^{\frac{3}{2}} + C$$

The population is $N(t) = 500t + \frac{2}{3}t^{\frac{3}{2}} + C$

5. The amount of money saved is $\int \frac{dS}{dt}dt$.

$$\begin{aligned} &\int\left(2.1t^2 - 65.4t + 491.6\right)dt \\ &= 2.1\left(\frac{t^3}{3}\right) - 65.4\left(\frac{t^2}{2}\right) + 491.6t + C \\ &= 0.7t^3 - 32.7t^2 + 491.6t + C \end{aligned}$$

The amount of money saved is $S(t) = 0.7t^3 - 32.7t^2 + 491.6t + C$

Problems 14.2

1. $\int 7\,dx = 7x + C$

2. $\int \frac{1}{2x}dx = \frac{1}{2}\int\frac{1}{x}dx = \frac{1}{2}\ln|x| + C$

3. $\int x^8 dx = \frac{x^{8+1}}{8+1} + C = \frac{x^9}{9} + C$

4. $\int 5x^{24}\,dx = 5\int x^{24}\,dx = 5\cdot\frac{x^{24+1}}{24+1} + C = 5\cdot\frac{x^{25}}{25} + C = \frac{x^{25}}{5} + C$

5. $\int 5x^{-7}dx = 5\int x^{-7}dx = 5\cdot\frac{x^{-7+1}}{-7+1} + C = 5\cdot\frac{x^{-6}}{-6} + C = -\frac{5}{6x^6} + C$

6. $\int\frac{z^{-3}}{3}dz = \frac{1}{3}\int z^{-3}dz = \frac{1}{3}\cdot\frac{z^{-3+1}}{-3+1} + C = \frac{1}{3}\cdot\frac{z^{-2}}{-2} + C = -\frac{1}{6z^2} + C$

7. $\int\frac{2}{x^{10}}dx = 2\int x^{-10}dx = 2\cdot\frac{x^{-10+1}}{-10+1} + C = \frac{2x^{-9}}{-9} + C = -\frac{2}{9x^9} + C$

8. $\int\frac{7}{x^4}dx = 7\int x^{-4}dx = 7\cdot\frac{x^{-4+1}}{-4+1} + C = \frac{7x^{-3}}{-3} + C = -\frac{7}{3x^3} + C$

9. $\int\frac{1}{t^{7/4}}dt = \int t^{-7/4}dt = \frac{t^{-7/4+1}}{-\frac{7}{4}+1} + C = \frac{t^{-3/4}}{-\frac{3}{4}} + C = -\frac{4}{3t^{3/4}} + C$

10. $\int\frac{7}{2x^{\frac{9}{4}}}dx = \frac{7}{2}\int x^{-\frac{9}{4}}dx = \frac{7}{2}\cdot\frac{x^{-\frac{9}{4}+1}}{-\frac{9}{4}+1} + C = \frac{7}{2}\cdot\frac{x^{-\frac{5}{4}}}{-\frac{5}{4}} + C = -\frac{14}{5x^{\frac{5}{4}}} + C$

11. $\int (4+t)dt = \int 4\,dt + \int t\,dt = 4t + \frac{t^{1+1}}{1+1} + C$

$= 4t + \frac{t^2}{2} + C$

12. $\int \left(r^3 + 2r\right)dr = \int r^3 + 2\int r\,dr$

$= \frac{r^{3+1}}{3+1} + 2\cdot\frac{r^{1+1}}{1+1} + C$

$= \frac{r^4}{4} + r^2 + C$

13. $\int \left(y^5 - 5y\right)dy = \int y^5 dy - \int 5y\,dy$

$= \frac{y^{5+1}}{5+1} - 5\cdot\frac{y^{1+1}}{1+1} + C$

$= \frac{y^6}{6} - 5\cdot\frac{y^2}{2} + C = \frac{y^6}{6} - \frac{5y^2}{2} + C$

14. $\int \left(5 - 2w - 6w^2\right)dw$

$= \int 5\,dw - 2\int w\,dw - 6\int w^2 dw$

$= 5w - 2\cdot\frac{w^2}{2} - 6\cdot\frac{w^3}{3} + C$

$= 5w - w^2 - 2w^3 + C$

15. $\int \left(3t^2 - 4t + 5\right)dt = 3\int t^2 dt - 4\int t\,dt + \int 5\,dt$

$= 3\cdot\frac{t^3}{3} - 4\cdot\frac{t^2}{2} + 5t + C = t^3 - 2t^2 + 5t + C$

16. $\int (1 + t^2 + t^4 + t^6)dt$

$= \int 1\,dt + \int t^2\,dt + \int t^4\,dt + \int t^6\,dt$

$= t + \frac{t^3}{3} + \frac{t^5}{5} + \frac{t^7}{7} + C$

17. Since $7 + e$ is a constant,
$\int (7+e)dx = (7+e)x + C$.

18. $\int \left(5 - 2^{-1}\right)dx = \int \left(5 - \frac{1}{2}\right)dx = \int \frac{9}{2}dx = \frac{9}{2}x + C$

19. $\int \left(\frac{x}{7} - \frac{3}{4}x^4\right)dx = \frac{1}{7}\int x\,dx - \frac{3}{4}\int x^4 dx$

$= \frac{1}{7}\cdot\frac{x^2}{2} - \frac{3}{4}\cdot\frac{x^5}{5} + C$

$= \frac{x^2}{14} - \frac{3x^5}{20} + C$

20. $\int \left(\frac{2x^2}{7} - \frac{8}{3}x^4\right)dx = \frac{2}{7}\int x^2 dx - \frac{8}{3}\int x^4 dx$

$= \frac{2}{7}\cdot\frac{x^3}{3} - \frac{8}{3}\cdot\frac{x^5}{5} + C$

$= \frac{2x^3}{21} - \frac{8x^5}{15} + C$

21. $\int \pi e^x dx = \pi\int e^x dx = \pi e^x + C$

22. $\int \left(\frac{e^x}{3} + 2x\right)dx = \frac{1}{3}\int e^x dx + 2\int x\,dx$

$= \frac{1}{3}e^x + 2\cdot\frac{x^2}{2} + C$

$= \frac{e^x}{3} + x^2 + C$

23. $\int \left(x^{8.3} - 9x^6 + 3x^{-4} + x^{-3}\right)dx$

$= \frac{x^{9.3}}{9.3} - 9\cdot\frac{x^7}{7} + 3\cdot\frac{x^{-3}}{-3} + \frac{x^{-2}}{-2} + C$

$= \frac{x^{9.3}}{9.3} - \frac{9x^7}{7} - \frac{1}{x^3} - \frac{1}{2x^2} + C$

24. $\int (0.7y^3 + 10 + 2y^{-3})dy$

$= 0.7\cdot\frac{y^4}{4} + 10y + 2\cdot\frac{y^{-2}}{-2} + C$

$= 0.175y^4 + 10y - \frac{1}{y^2} + C$

25. $\int \frac{-2\sqrt{x}}{3}dx = -\frac{2}{3}\int x^{\frac{1}{2}}dx = -\frac{2}{3}\cdot\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C$

$= -\frac{2}{3}\cdot\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + C = -\frac{4x^{\frac{3}{2}}}{9} + C$

26. $\int dz = \int 1\,dz = 1\cdot z + C = z + C$

27. $\int \frac{1}{4\sqrt[8]{x^2}}dx = \frac{1}{4}\int x^{-\frac{1}{4}}dx = \frac{1}{4}\cdot\frac{x^{-\frac{1}{4}+1}}{-\frac{1}{4}+1} + C$

$= \frac{1}{4}\cdot\frac{x^{\frac{3}{4}}}{\frac{3}{4}} + C = \frac{x^{\frac{3}{4}}}{3} + C$

28. $\int \frac{-4}{(3x)^3}dx = \int \frac{-4}{27x^3}dx = -\frac{4}{27}\int x^{-3}\,dx$

$= -\frac{4}{27}\cdot\frac{x^{-3+1}}{-3+1} + C$

$= -\frac{4}{27}\cdot\frac{x^{-2}}{-2} + C = \frac{2}{27x^2} + C$

29. $\int \left(\frac{x^3}{3} - \frac{3}{x^3}\right)dx = \frac{1}{3}\int x^3 dx - 3\int x^{-3} dx$

$= \frac{1}{3}\cdot\frac{x^{3+1}}{3+1} - 3\cdot\frac{x^{-3+1}}{-3+1} + C$

$= \frac{1}{3}\cdot\frac{x^4}{4} - 3\cdot\frac{x^{-2}}{-2} + C = \frac{x^4}{12} + \frac{3}{2x^2} + C$

30. $\int \left(\frac{1}{2x^3} - \frac{1}{x^4}\right)dx = \frac{1}{2}\int x^{-3}dx - \int x^{-4}dx$

$= \frac{1}{2}\cdot\frac{x^{-2}}{-2} - \frac{x^{-3}}{-3} + C$

$= -\frac{1}{4x^2} + \frac{1}{3x^3} + C$

31. $\int \left(\frac{3w^2}{2} - \frac{2}{3w^2}\right)dw = \frac{3}{2}\int w^2 dw - \frac{2}{3}\int w^{-2} dw$

$= \frac{3}{2}\cdot\frac{w^3}{3} - \frac{2}{3}\cdot\frac{w^{-1}}{-1} + C = \frac{w^3}{2} + \frac{2}{3w} + C$

32. $\int \frac{4}{e^{-s}}ds = 4\int e^s ds = 4e^s + C$

33. $\int \frac{3u-4}{5}du = \frac{1}{5}\int (3u-4)du = \frac{1}{5}\left(3\int u\,du - 4\int du\right)$

$= \frac{1}{5}\left(3\frac{u^2}{2} - 4u\right) + C = \frac{3}{10}u^2 - \frac{4}{5}u + C$

$= \frac{1}{7}\left(2\int z\,dz - \int 5\,dz\right)$

$= \frac{1}{7}\left(2\cdot\frac{z^2}{2} - 5z\right) + C = \frac{1}{7}\left(z^2 - 5z\right) + C$

34. $\int \frac{1}{12}\left(\frac{1}{3}e^x\right)dx = \int \frac{1}{36}e^x dx$

$= \frac{1}{36}\int e^x dx = \frac{1}{36}e^x + C$

35. $\int (u^e + e^u)du = \int u^e\,du + \int e^u\,du$

$= \frac{u^{e+1}}{e+1} + e^u + C$

36. $\int \left(3y^3 - 2y^2 + \frac{e^y}{6}\right)dy$

$= 3\int y^3 dy - 2\int y^2 dy + \frac{1}{6}\int e^y dy$

$= 3\cdot\frac{y^4}{4} - 2\cdot\frac{y^3}{3} + \frac{1}{6}\cdot e^y + C$

$= \frac{3y^4}{4} - \frac{2y^3}{3} + \frac{e^y}{6} + C$

37. $\int \left(2\sqrt{x} - 3\sqrt[4]{x}\right)dx = \int \left(2x^{\frac{1}{2}} - 3x^{\frac{1}{4}}\right)dx$

$= 2\int x^{\frac{1}{2}}dx - 3\int x^{\frac{1}{4}}dx$

$= 2\cdot\frac{x^{\frac{3}{2}}}{\frac{3}{2}} - 3\cdot\frac{x^{\frac{5}{4}}}{\frac{5}{4}} + C = \frac{4x^{\frac{3}{2}}}{3} - \frac{12x^{\frac{5}{4}}}{5} + C$

38. $\int 0\,dt = 0\cdot t + C = C$

39. $\int\left(-\frac{\sqrt[3]{x^2}}{5}-\frac{7}{2\sqrt{x}}+6x\right)dx$

$=\int\left(-\frac{x^{\frac{2}{3}}}{5}-\frac{7x^{-\frac{1}{2}}}{2}+6x\right)dx$

$=-\frac{1}{5}\int x^{\frac{2}{3}}dx-\frac{7}{2}\int x^{-\frac{1}{2}}dx+6\int x\,dx$

$=-\frac{1}{5}\cdot\frac{x^{\frac{5}{3}}}{\frac{5}{3}}-\frac{7}{2}\cdot\frac{x^{\frac{1}{2}}}{\frac{1}{2}}+6\cdot\frac{x^2}{2}+C$

$=-\frac{3x^{\frac{5}{3}}}{25}-7x^{\frac{1}{2}}+3x^2+C$

40. $\int\left(\sqrt[3]{u}+\frac{1}{\sqrt{u}}\right)du=\int\left(u^{\frac{1}{3}}+u^{-\frac{1}{2}}\right)du$

$=\int u^{\frac{1}{3}}du+\int u^{-\frac{1}{2}}du$

$=\frac{u^{\frac{4}{3}}}{\frac{4}{3}}+\frac{u^{\frac{1}{2}}}{\frac{1}{2}}+C$

$=\frac{3u^{\frac{4}{3}}}{4}+2u^{\frac{1}{2}}+C$

41. $\int\left(x^2+5\right)(x-3)dx=\int\left(x^3-3x^2+5x-15\right)dx$

$=\frac{x^4}{4}-3\cdot\frac{x^3}{3}+5\cdot\frac{x^2}{2}-15x+C$

$=\frac{x^4}{4}-x^3+\frac{5x^2}{2}-15x+C$

42. $\int x^4\left(x^3+8x^2+7\right)dx=\int\left(x^7+8x^6+7x^4\right)dx$

$=\frac{x^8}{8}+8\cdot\frac{x^7}{7}+7\cdot\frac{x^5}{5}+C$

$=\frac{x^8}{8}+\frac{8x^7}{7}+\frac{7x^5}{5}+C$

43. $\int\sqrt{x}(x+3)dx=\int\left(x^{\frac{3}{2}}+3x^{\frac{1}{2}}\right)dx$

$=\frac{x^{\frac{5}{2}}}{\frac{5}{2}}+3\cdot\frac{x^{\frac{3}{2}}}{\frac{3}{2}}+C$

$=\frac{2x^{\frac{5}{2}}}{5}+2x^{\frac{3}{2}}+C$

44. $\int(z+2)^2dz=\int\left(z^2+4z+4\right)dz$

$=\frac{z^3}{3}+4\cdot\frac{z^2}{2}+4z+C$

$=\frac{z^3}{3}+2z^2+4z+C$

45. $\int(3u+2)^3du=\int(27u^3+54u^2+36u+8)du$

$=27\cdot\frac{u^4}{4}+54\cdot\frac{u^3}{3}+36\cdot\frac{u^2}{2}+8u+C$

$=\frac{27}{4}u^4+18u^3+18u^2+8u+C$

46. $\int\left(\frac{2}{\sqrt[5]{x}}-1\right)^2dx=\int\left(2x^{-\frac{1}{5}}-1\right)^2dx$

$=\int\left(4x^{-\frac{2}{5}}-4x^{-\frac{1}{5}}+1\right)dx$

$=4\cdot\frac{x^{\frac{3}{5}}}{\frac{3}{5}}-4\cdot\frac{x^{\frac{4}{5}}}{\frac{4}{5}}+x+C$

$=\frac{20x^{\frac{3}{5}}}{3}-5x^{\frac{4}{5}}+x+C$

47. $\int v^{-2}\left(2v^4+3v^2-2v^{-3}\right)dv$

$=\int\left(2v^2+3-2v^{-5}\right)dv$

$=2\cdot\frac{v^3}{3}+3v-2\cdot\frac{v^{-4}}{-4}+C$

$=\frac{2v^3}{3}+3v+\frac{1}{2v^4}+C$

48. $\int\left[6e^u-u^3\left(\sqrt{u}+1\right)\right]du=\int\left[6e^u-u^{\frac{7}{2}}-u^3\right]du$

$=6\cdot e^u-\frac{u^{\frac{9}{2}}}{\frac{9}{2}}-\frac{u^4}{4}+C$

$=6e^u-\frac{2u^{\frac{9}{2}}}{9}-\frac{u^4}{4}+C$

49. $\int\frac{z^4+10z^3}{2z^2}dz=\frac{1}{2}\int\left(\frac{z^4}{z^2}+\frac{10z^3}{z^2}\right)dz$

$=\frac{1}{2}\int\left(z^2+10z\right)dz$

$=\frac{1}{2}\left(\frac{z^3}{3}+10\cdot\frac{z^2}{2}\right)+C$

$=\frac{z^3}{6}+\frac{5z^2}{2}+C$

50. $\int\frac{x^4-5x^2+2x}{5x^2}dx=\frac{1}{5}\int\left(x^2-5+\frac{2}{x}\right)dx$

$=\frac{1}{5}\left(\frac{x^3}{3}-5x+2\ln|x|\right)+C$

51. $\int\frac{e^x+e^{2x}}{e^x}dx=\int\left(\frac{e^x}{e^x}+\frac{e^{2x}}{e^x}\right)dx$

$=\int\left(1+e^x\right)dx$

$=x+e^x+C$

52. $\int\frac{(x^3+1)^2}{x^2}dx=\int\frac{x^6+2x^3+1}{x^2}dx$

$=\int(x^4+2x+x^{-2})dx$

$=\frac{x^5}{5}+2\cdot\frac{x^2}{2}+\frac{x^{-1}}{-1}+C$

$=\frac{x^5}{5}+x^2-\frac{1}{x}+C$

53. No, $F(x)-G(x)$ might be a nonzero constant.

54. a. $F(x)=\frac{d}{dx}\left(xe^x\right)=xe^x+e^x(1)=e^x(x+1)$

b. There is only one.

55. Because an antiderivative of the derivative of a function is the function itself, we have

$\int\frac{d}{dx}\left[\frac{1}{\sqrt{x^2+1}}\right]dx=\frac{1}{\sqrt{x^2+1}}+C$.

Principles in Practice 14.3

1. $N(t)=\int\frac{dN}{dt}dt=\int\left(800+200e^t\right)dt$

$=800t+200e^t+C$

Since $N(5)=40{,}000$, we have

$40{,}000=800(5)+200e^5+C$, so

$C=40{,}000-\left(4000+200e^5\right)$

$=36{,}000-200e^5\approx 6317.37$

$N(t)=800t+200e^t+6317.37$

2. Since $y''=\frac{d}{dt}(y')=84t+24$

$y'=\int(84t+24)dt=84\left(\frac{t^2}{2}\right)+24t+C_1$

$=42t^2+24t+C_1$

Since $y'(8)=2891$, we have

$2891=42(8)^2+24(8)+C_1=2880+C_1$, so

$C_1=2891-2880=11$, and $y'=42t^2+24t+11$.

$y(t)=\int y'dt=\int\left(42t^2+24t+11\right)dt$

$=42\left(\frac{t^3}{3}\right)+24\left(\frac{t^2}{2}\right)+11t+C_2$

$=14t^3+12t^2+11t+C_2$

Since $y(2)=185$, we have

$185=14(2)^3+12(2)^2+11(2)+C_2$

$=182+C_2$, so $C_2=185-182=3$.

$y(t)=14t^3+12t^2+11t+3$