$$\Delta V \approx 4\pi\left(6.5\times10^{-4}\right)^2\left(10^{-5}\right)=\left(1.69\times10^{-11}\right)\pi \text{ cm}^3.$$

44. $(P+a)(v+b)=k$

$$P=\frac{k}{v+b}-a$$

$$dP=-k(v+b)^{-2}dv$$

45. a. We substitute $q = 40$ and $p = 20$

$$2+\frac{40^2}{200}=\frac{4000}{20^2}$$
$$2+8=10$$
$$10=10$$

b. We differentiate implicitly with respect to p.

$$0+\frac{1}{200}\left(2q\frac{dq}{dp}\right)=-\frac{8000}{p^3}$$

From part (a) $q = 40$ when $p = 20$. Substituting gives

$$\frac{1}{200}\left(2\cdot40\frac{dq}{dp}\right)=-\frac{8000}{20^3}$$
$$\frac{dq}{dp}=-2.5$$

c. $q(p+dp)\approx q(p)+dq=q(p)+q'(p)dp$

$$q(19.20)=q(20+(-0.8))$$
$$\approx q(20)+q'(20)dp$$
$$=40+(-2.5)(-0.8)$$
$$=42 \text{ units}$$

46. a. Profit $= TR - TC = pq - \bar{c}q$

$$P=\frac{1}{2}q^3-66q^2+7000q-\left(500q-q^2+\frac{80,000}{2}\right)=\frac{1}{2}q^3-65q^2+6500q-40,000$$

If $q = 100$, then $P=\frac{1}{2}(100)^3-65(100)^2+6500(100)-40,000=460,000$

b. We use $P(q + dq) \approx P(q) + dP$ with $q = 100$ and $dq = -2$.

$$P(98)=P(100+(-2))$$
$$\approx P(100)+\left(\frac{3}{2}q^2-130q+6500\right)dq$$
$$=460,000+\left[\frac{3}{2}(100)^2-130(100)+6500\right](-2)$$
$$=\$443,000$$

Principles in Practice 14.2

1. $\int 28.3\,dq=28.3q+C$

The form of the cost function is $28.3q + C$.

2. $\int 0.12t^2dt=0.12\frac{t^3}{3}+C=0.04t^3+C$

The form of the revenue function is $R(t)=0.04t^3+C$.

3. Let $S(t)$ = the number of subscribers t months after the competition entered the market, then $S'(t)=-\frac{480}{t^3}$.

$$S(t)=\int-\frac{480}{t^3}dt=-480\int t^{-3}dt$$
$$=-480\left(\frac{t^{-2}}{-2}\right)+C=240t^{-2}+C=\frac{240}{t^2}+C$$

The number of subscribers is $S(t)=\frac{240}{t^2}+C$.

4. $\int\left(500+300\sqrt{t}\right)dt=\int\left(500+300t^{\frac{1}{2}}\right)dt$

$$=500t+\frac{t^{\frac{3}{2}}}{\frac{3}{2}}+C=500t+\frac{2}{3}t^{\frac{3}{2}}+C$$

The population is $N(t)=500t+\frac{2}{3}t^{\frac{3}{2}}+C$

5. The amount of money saved is $\int\frac{dS}{dt}dt$.

$$\int\left(2.1t^2-65.4t+491.6\right)dt$$
$$=2.1\left(\frac{t^3}{3}\right)-65.4\left(\frac{t^2}{2}\right)+491.6t+C$$
$$=0.7t^3-32.7t^2+491.6t+C$$

The amount of money saved is $S(t)=0.7t^3-32.7t^2+491.6t+C$

Problems 14.2

1. $\int 7\,dx=7x+C$

2. $\int\frac{1}{2x}dx=\frac{1}{2}\int\frac{1}{x}dx=\frac{1}{2}\ln|x|+C$

3. $\int x^8dx=\frac{x^{8+1}}{8+1}+C=\frac{x^9}{9}+C$

4. $\int 5x^{24}\,dx=5\int x^{24}\,dx=5\cdot\frac{x^{24+1}}{24+1}+C$

$$=5\cdot\frac{x^{25}}{25}+C=\frac{x^{25}}{5}+C$$

5. $\int 5x^{-7}dx=5\int x^{-7}dx=5\cdot\frac{x^{-7+1}}{-7+1}+C$

$$=5\cdot\frac{x^{-6}}{-6}+C=-\frac{5}{6x^6}+C$$

6. $\int\frac{z^{-3}}{3}dz=\frac{1}{3}\int z^{-3}dz=\frac{1}{3}\cdot\frac{z^{-3+1}}{-3+1}+C$

$$=\frac{1}{3}\cdot\frac{z^{-2}}{-2}+C=-\frac{1}{6z^2}+C$$

7. $\int\frac{2}{x^{10}}dx=2\int x^{-10}dx=2\cdot\frac{x^{-10+1}}{-10+1}+C$

$$=\frac{2x^{-9}}{-9}+C=-\frac{2}{9x^9}+C$$

8. $\int\frac{7}{x^4}dx=7\int x^{-4}dx=7\cdot\frac{x^{-4+1}}{-4+1}+C=\frac{7x^{-3}}{-3}+C$

$$=-\frac{7}{3x^3}+C$$

9. $\int\frac{1}{t^{7/4}}dt=\int t^{-7/4}dt=\frac{t^{-7/4+1}}{-\frac{7}{4}+1}+C=\frac{t^{-3/4}}{-\frac{3}{4}}+C$

$$=-\frac{4}{3t^{3/4}}+C$$

10. $\int\frac{7}{2x^{\frac{9}{4}}}dx=\frac{7}{2}\int x^{-\frac{9}{4}}dx=\frac{7}{2}\cdot\frac{x^{-\frac{9}{4}+1}}{-\frac{9}{4}+1}+C$

$$=\frac{7}{2}\cdot\frac{x^{-\frac{5}{4}}}{-\frac{5}{4}}+C$$
$$=-\frac{14}{5x^{\frac{5}{4}}}+C$$

11. $\int(4+t)dt = \int 4\,dt + \int t\,dt = 4t + \frac{t^{t+1}}{1+1} + C$
$= 4t + \frac{t^2}{2} + C$

12. $\int\left(r^3+2r\right)dr = \int r^3 + 2\int r\,dr$
$= \frac{r^{3+1}}{3+1} + 2\cdot\frac{r^{1+1}}{1+1} + C$
$= \frac{r^4}{4} + r^2 + C$

13. $\int\left(y^5-5y\right)dy = \int y^5 dy - \int 5y\,dy$
$= \frac{y^{5+1}}{5+1} - 5\cdot\frac{y^{1+1}}{1+1} + C$
$= \frac{y^6}{6} - 5\cdot\frac{y^2}{2} + C = \frac{y^6}{6} - \frac{5y^2}{2} + C$

14. $\int\left(5-2w-6w^2\right)dw$
$= \int 5\,dw - 2\int w\,dw - 6\int w^2 dw$
$= 5w - 2\cdot\frac{w^2}{2} - 6\cdot\frac{w^3}{3} + C$
$= 5w - w^2 - 2w^3 + C$

15. $\int\left(3t^2-4t+5\right)dt = 3\int t^2 dt - 4\int t\,dt + \int 5\,dt$
$= 3\cdot\frac{t^3}{3} - 4\cdot\frac{t^2}{2} + 5t + C = t^3 - 2t^2 + 5t + C$

16. $\int(1+t^2+t^4+t^6)dt$
$= \int 1\,dt + \int t^2\,dt + \int t^4\,dt + \int t^6\,dt$
$= t + \frac{t^3}{3} + \frac{t^5}{5} + \frac{t^7}{7} + C$

17. Since $7 + e$ is a constant,
$\int(7+e)dx = (7+e)x + C$.

18. $\int\left(5-2^{-1}\right)dx = \int\left(5-\frac{1}{2}\right)dx = \int\frac{9}{2}dx = \frac{9}{2}x + C$

19. $\int\left(\frac{x}{7}-\frac{3}{4}x^4\right)dx = \frac{1}{7}\int x\,dx - \frac{3}{4}\int x^4 dx$
$= \frac{1}{7}\cdot\frac{x^2}{2} - \frac{3}{4}\cdot\frac{x^5}{5} + C$
$= \frac{x^2}{14} - \frac{3x^5}{20} + C$

20. $\int\left(\frac{2x^2}{7}-\frac{8}{3}x^4\right)dx = \frac{2}{7}\int x^2 dx - \frac{8}{3}\int x^4 dx$
$= \frac{2}{7}\cdot\frac{x^3}{3} - \frac{8}{3}\cdot\frac{x^5}{5} + C$
$= \frac{2x^3}{21} - \frac{8x^5}{15} + C$

21. $\int \pi e^x dx = \pi\int e^x dx = \pi e^x + C$

22. $\int\left(\frac{e^x}{3}+2x\right)dx = \frac{1}{3}\int e^x dx + 2\int x\,dx$
$= \frac{1}{3}e^x + 2\cdot\frac{x^2}{2} + C$
$= \frac{e^x}{3} + x^2 + C$

23. $\int\left(x^{8.3}-9x^6+3x^{-4}+x^{-3}\right)dx$
$= \frac{x^{9.3}}{9.3} - 9\cdot\frac{x^7}{7} + 3\cdot\frac{x^{-3}}{-3} + \frac{x^{-2}}{-2} + C$
$= \frac{x^{9.3}}{9.3} - \frac{9x^7}{7} - \frac{1}{x^3} - \frac{1}{2x^2} + C$

24. $\int(0.7y^3+10+2y^{-3})dy$
$= 0.7\cdot\frac{y^4}{4} + 10y + 2\cdot\frac{y^{-2}}{-2} + C$
$= 0.175y^4 + 10y - \frac{1}{y^2} + C$

25. $\int\frac{-2\sqrt{x}}{3}dx = -\frac{2}{3}\int x^{\frac{1}{2}}dx = -\frac{2}{3}\cdot\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C$
$= -\frac{2}{3}\cdot\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + C = -\frac{4x^{\frac{3}{2}}}{9} + C$

26. $\int dz = \int 1\,dz = 1\cdot z + C = z + C$

27. $\int\frac{1}{4\sqrt[8]{x^2}}dx = \frac{1}{4}\int x^{-\frac{1}{4}}dx = \frac{1}{4}\cdot\frac{x^{-\frac{1}{4}+1}}{-\frac{1}{4}+1} + C$
$= \frac{1}{4}\cdot\frac{x^{\frac{3}{4}}}{\frac{3}{4}} + C = \frac{x^{\frac{3}{4}}}{3} + C$

28. $\int\frac{-4}{(3x)^3}dx = \int\frac{-4}{27x^3}dx = -\frac{4}{27}\int x^{-3}\,dx$
$= -\frac{4}{27}\cdot\frac{x^{-3+1}}{-3+1} + C$
$= -\frac{4}{27}\cdot\frac{x^{-2}}{-2} + C = \frac{2}{27x^2} + C$

29. $\int\left(\frac{x^3}{3}-\frac{3}{x^3}\right)dx = \frac{1}{3}\int x^3 dx - 3\int x^{-3}dx$
$= \frac{1}{3}\cdot\frac{x^{3+1}}{3+1} - 3\cdot\frac{x^{-3+1}}{-3+1} + C$
$= \frac{1}{3}\cdot\frac{x^4}{4} - 3\cdot\frac{x^{-2}}{-2} + C = \frac{x^4}{12} + \frac{3}{2x^2} + C$

30. $\int\left(\frac{1}{2x^3}-\frac{1}{x^4}\right)dx = \frac{1}{2}\int x^{-3}dx - \int x^{-4}dx$
$= \frac{1}{2}\cdot\frac{x^{-2}}{-2} - \frac{x^{-3}}{-3} + C$
$= -\frac{1}{4x^2} + \frac{1}{3x^3} + C$

31. $\int\left(\frac{3w^2}{2}-\frac{2}{3w^2}\right)dw = \frac{3}{2}\int w^2 dw - \frac{2}{3}\int w^{-2}dw$
$= \frac{3}{2}\cdot\frac{w^3}{3} - \frac{2}{3}\cdot\frac{w^{-1}}{-1} + C = \frac{w^3}{2} + \frac{2}{3w} + C$

32. $\int\frac{4}{e^{-s}}ds = 4\int e^s ds = 4e^s + C$

33. $\int\frac{3u-4}{5}du = \frac{1}{5}\int(3u-4)du = \frac{1}{5}\left(3\int u\,du - 4\int du\right)$
$= \frac{1}{5}\left(3\frac{u^2}{2} - 4u\right) + C = \frac{3}{10}u^2 - \frac{4}{5}u + C$
$= \frac{1}{7}\left(2\int z\,dz - \int 5\,dz\right)$
$= \frac{1}{7}\left(2\cdot\frac{z^2}{2} - 5z\right) + C = \frac{1}{7}\left(z^2-5z\right) + C$

34. $\int\frac{1}{12}\left(\frac{1}{3}e^x\right)dx = \int\frac{1}{36}e^x dx$
$= \frac{1}{36}\int e^x dx = \frac{1}{36}e^x + C$

35. $\int(u^e+e^u)du = \int u^e\,du + \int e^u\,du$
$= \frac{u^{e+1}}{e+1} + e^u + C$

36. $\int\left(3y^3-2y^2+\frac{e^y}{6}\right)dy$
$= 3\int y^3 dy - 2\int y^2 dy + \frac{1}{6}\int e^y dy$
$= 3\cdot\frac{y^4}{4} - 2\cdot\frac{y^3}{3} + \frac{1}{6}\cdot e^y + C$
$= \frac{3y^4}{4} - \frac{2y^3}{3} + \frac{e^y}{6} + C$

37. $\int\left(2\sqrt{x}-3\sqrt[4]{x}\right)dx = \int\left(2x^{\frac{1}{2}}-3x^{\frac{1}{4}}\right)dx$
$= 2\int x^{\frac{1}{2}}dx - 3\int x^{\frac{1}{4}}dx$
$= 2\cdot\frac{x^{\frac{3}{2}}}{\frac{3}{2}} - 3\cdot\frac{x^{\frac{5}{4}}}{\frac{5}{4}} + C = \frac{4x^{\frac{3}{2}}}{3} - \frac{12x^{\frac{5}{4}}}{5} + C$

38. $\int 0\,dt = 0\cdot t + C = C$

39. $\int\left(-\frac{\sqrt[3]{x^2}}{5}-\frac{7}{2\sqrt{x}}+6x\right)dx$

$=\int\left(-\frac{x^{\frac{2}{3}}}{5}-\frac{7x^{-\frac{1}{2}}}{2}+6x\right)dx$

$=-\frac{1}{5}\int x^{\frac{2}{3}}dx-\frac{7}{2}\int x^{-\frac{1}{2}}dx+6\int x\,dx$

$=-\frac{1}{5}\cdot\frac{x^{\frac{5}{3}}}{\frac{5}{3}}-\frac{7}{2}\cdot\frac{x^{\frac{1}{2}}}{\frac{1}{2}}+6\cdot\frac{x^2}{2}+C$

$=-\frac{3x^{\frac{5}{3}}}{25}-7x^{\frac{1}{2}}+3x^2+C$

40. $\int\left(\sqrt[3]{u}+\frac{1}{\sqrt{u}}\right)du=\int\left(u^{\frac{1}{3}}+u^{-\frac{1}{2}}\right)du$

$=\int u^{\frac{1}{3}}du+\int u^{-\frac{1}{2}}du$

$=\frac{u^{\frac{4}{3}}}{\frac{4}{3}}+\frac{u^{\frac{1}{2}}}{\frac{1}{2}}+C$

$=\frac{3u^{\frac{4}{3}}}{4}+2u^{\frac{1}{2}}+C$

41. $\int\left(x^2+5\right)(x-3)dx=\int\left(x^3-3x^2+5x-15\right)dx$

$=\frac{x^4}{4}-3\cdot\frac{x^3}{3}+5\cdot\frac{x^2}{2}-15x+C$

$=\frac{x^4}{4}-x^3+\frac{5x^2}{2}-15x+C$

42. $\int x^4\left(x^3+8x^2+7\right)dx=\int\left(x^7+8x^6+7x^4\right)dx$

$=\frac{x^8}{8}+8\cdot\frac{x^7}{7}+7\cdot\frac{x^5}{5}+C$

$=\frac{x^8}{8}+\frac{8x^7}{7}+\frac{7x^5}{5}+C$

43. $\int\sqrt{x}(x+3)dx=\int\left(x^{\frac{3}{2}}+3x^{\frac{1}{2}}\right)dx$

$=\frac{x^{\frac{5}{2}}}{\frac{5}{2}}+3\cdot\frac{x^{\frac{3}{2}}}{\frac{3}{2}}+C$

$=\frac{2x^{\frac{5}{2}}}{5}+2x^{\frac{3}{2}}+C$

44. $\int(z+2)^2dz=\int\left(z^2+4z+4\right)dz$

$=\frac{z^3}{3}+4\cdot\frac{z^2}{2}+4z+C$

$=\frac{z^3}{3}+2z^2+4z+C$

45. $\int(3u+2)^3du=\int(27u^3+54u^2+36u+8)du$

$=27\cdot\frac{u^4}{4}+54\cdot\frac{u^3}{3}+36\cdot\frac{u^2}{2}+8u+C$

$=\frac{27}{4}u^4+18u^3+18u^2+8u+C$

46. $\int\left(\frac{2}{\sqrt[5]{x}}-1\right)^2dx=\int\left(2x^{-\frac{1}{5}}-1\right)^2dx$

$=\int\left(4x^{-\frac{2}{5}}-4x^{-\frac{1}{5}}+1\right)dx$

$=4\cdot\frac{x^{\frac{3}{5}}}{\frac{3}{5}}-4\cdot\frac{x^{\frac{4}{5}}}{\frac{4}{5}}+x+C$

$=\frac{20x^{\frac{3}{5}}}{3}-5x^{\frac{4}{5}}+x+C$

47. $\int v^{-2}\left(2v^4+3v^2-2v^{-3}\right)dv$

$=\int\left(2v^2+3-2v^{-5}\right)dv$

$=2\cdot\frac{v^3}{3}+3v-2\cdot\frac{v^{-4}}{-4}+C$

$=\frac{2v^3}{3}+3v+\frac{1}{2v^4}+C$

48. $\int\left[6e^u-u^3\left(\sqrt{u}+1\right)\right]du=\int\left[6e^u-u^{\frac{7}{2}}-u^3\right]du$

$=6\cdot e^u-\frac{u^{\frac{9}{2}}}{\frac{9}{2}}-\frac{u^4}{4}+C$

$=6e^u-\frac{2u^{\frac{9}{2}}}{9}-\frac{u^4}{4}+C$

49. $\int\frac{z^4+10z^3}{2z^2}dz=\frac{1}{2}\int\left(\frac{z^4}{z^2}+\frac{10z^3}{z^2}\right)dz$

$=\frac{1}{2}\int\left(z^2+10z\right)dz$

$=\frac{1}{2}\left(\frac{z^3}{3}+10\cdot\frac{z^2}{2}\right)+C$

$=\frac{z^3}{6}+\frac{5z^2}{2}+C$

50. $\int\frac{x^4-5x^2+2x}{5x^2}dx=\frac{1}{5}\int\left(x^2-5+\frac{2}{x}\right)dx$

$=\frac{1}{5}\left(\frac{x^3}{3}-5x+2\ln|x|\right)+C$

51. $\int\frac{e^x+e^{2x}}{e^x}dx=\int\left(\frac{e^x}{e^x}+\frac{e^{2x}}{e^x}\right)dx$

$=\int\left(1+e^x\right)dx$

$=x+e^x+C$

52. $\int\frac{(x^3+1)^2}{x^2}dx=\int\frac{x^6+2x^3+1}{x^2}dx$

$=\int(x^4+2x+x^{-2})dx$

$=\frac{x^5}{5}+2\cdot\frac{x^2}{2}+\frac{x^{-1}}{-1}+C$

$=\frac{x^5}{5}+x^2-\frac{1}{x}+C$

53. No, $F(x)-G(x)$ might be a nonzero constant.

54. **a.** $F(x)=\frac{d}{dx}\left(xe^x\right)=xe^x+e^x(1)=e^x(x+1)$

b. There is only one.

55. Because an antiderivative of the derivative of a function is the function itself, we have

$\int\frac{d}{dx}\left[\frac{1}{\sqrt{x^2+1}}\right]dx=\frac{1}{\sqrt{x^2+1}}+C.$

Principles in Practice 14.3

1. $N(t)=\int\frac{dN}{dt}dt=\int\left(800+200e^t\right)dt$

$=800t+200e^t+C$

Since $N(5)=40{,}000$, we have

$40{,}000=800(5)+200e^5+C$, so

$C=40{,}000-\left(4000+200e^5\right)$

$=36{,}000-200e^5\approx6317.37$

$N(t)=800t+200e^t+6317.37$

2. Since $y''=\frac{d}{dt}(y')=84t+24$

$y'=\int(84t+24)dt=84\left(\frac{t^2}{2}\right)+24t+C_1$

$=42t^2+24t+C_1$

Since $y'(8)=2891$, we have

$2891=42(8)^2+24(8)+C_1=2880+C_1$, so

$C_1=2891-2880=11$, and $y'=42t^2+24t+11$.

$y(t)=\int y'dt=\int\left(42t^2+24t+11\right)dt$

$=42\left(\frac{t^3}{3}\right)+24\left(\frac{t^2}{2}\right)+11t+C_2$

$=14t^3+12t^2+11t+C_2$

Since $y(2)=185$, we have

$185=14(2)^3+12(2)^2+11(2)+C_2$

$=182+C_2$, so $C_2=185-182=3$.

$y(t)=14t^3+12t^2+11t+3$

Problems 14.3

1. $\frac{dy}{dx} = 3x - 4$

$y = \int (3x-4)dx = \frac{3x^2}{2} - 4x + C$

Using $y(-1) = \frac{13}{2}$ gives

$\frac{13}{2} = \frac{3(-1)^2}{2} - 4(-1) + C$

$\frac{13}{2} = \frac{11}{2} + C$

Thus $C = 1$, so $y = \frac{3x^2}{2} - 4x + 1$.

2. $\frac{dy}{dx} = x^2 - x$

$y = \int \left(x^2 - x\right)dx = \frac{x^3}{3} - \frac{x^2}{2} + C$

Using $y(3) = \frac{19}{2}$ gives $\frac{19}{2} = \frac{3^3}{3} - \frac{3^2}{2} + C$

$\frac{19}{2} = \frac{9}{2} + C$

Thus, $C = 5$, so $y = \frac{x^3}{3} - \frac{x^2}{2} + 5$.

3. $y' = \frac{5}{\sqrt{x}}$

$y = \int \frac{5}{\sqrt{x}}dx = \int 5x^{-\frac{1}{2}}dx = 5 \cdot \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C = 10\sqrt{x} + C$

$y(9) = 50$ implies $50 = 10\sqrt{9} + C$, $50 = 30 + C$, $C = 20$.

Thus $y = 10\sqrt{x} + 20$.

$y(16) = 10 \cdot 4 + 20 = 60$

4. $y' = -x^2 + 2x$

$y = \int \left(-x^2 + 2x\right)dx = -\frac{x^3}{3} + x^2 + C$

$y(2) = 1$ implies $1 = -\frac{8}{3} + 4 + C$, so $C = -\frac{1}{3}$.

Thus $y = -\frac{x^3}{3} + x^2 - \frac{1}{3}$.

$y(1) = -\frac{1}{3} + 1 - \frac{1}{3} = \frac{1}{3}$

5. $y'' = -3x^2 + 4x$

$y' = \int (-3x^2 + 4x)dx = -x^3 + 2x^2 + C_1$

$y'(1) = 2$ implies $2 = -1 + 2 + C_1$, so $C_1 = 1$.

$y = \int (-x^3 + 2x^2 + 1)dx = -\frac{x^4}{4} + \frac{2x^3}{3} + x + C_2$

$y(1) = 3$ implies $3 = -\frac{1}{4} + \frac{2}{3} + 1 + C_2$, so

$C_2 = \frac{19}{12}$. Thus $y = -\frac{x^4}{4} + \frac{2x^3}{3} + x + \frac{19}{12}$.

6. $y'' = x + 1$

$y' = \int (x+1)dx = \frac{x^2}{2} + x + C_1$

$y'(0) = 0$ implies $0 = 0 + 0 + C_1$, so $C_1 = 0$.

$y = \int \left[\frac{x^2}{2} + x\right]dx = \frac{x^3}{6} + \frac{x^2}{2} + C_2$.

$y(0) = 5$ implies $5 = 0 + 0 + C_2$, so $C_2 = 5$. Thus

$y = \frac{x^3}{6} + \frac{x^2}{2} + 5$.

7. $y''' = 2x$

$y'' = \int 2x\,dx = x^2 + C_1$

$y''(-1) = 3$ implies that $3 = 1 + C_1$, so $C_1 = 2$.

$y' = \int \left(x^2 + 2\right)dx = \frac{x^3}{3} + 2x + C_2$

$y'(3) = 10$ implies $10 = 9 + 6 + C_2$, so $C_2 = -5$.

$y = \int \left(\frac{x^3}{3} + 2x - 5\right)dx = \frac{x^4}{12} + x^2 - 5x + C_3$.

$y(0) = 13$ implies that $13 = 0 + 0 - 0 + C_3$, so

$C_3 = 13$. Therefore $y = \frac{x^4}{12} + x^2 - 5x + 13$.

8. $y''' = e^x + 1$

$y'' = \int \left(e^x + 1\right)dx = e^x + x + C_1$

$y''(0) = 1$ implies $1 = 1 + 0 + C_1$, so $C_1 = 0$.

$y' = \int \left(e^x + x\right)dx = e^x + \frac{x^2}{2} + C_2$

$y'(0) = 2$ implies $2 = 1 + 0 + C_2$, so $C_2 = 1$.

$y = \int \left[e^x + \frac{x^2}{2} + 1\right]dx = e^x + \frac{x^3}{6} + x + C_3$

$y(0) = 3$ implies that $3 = 1 + 0 + 0 + C_3$, so

$C_3 = 2$. Thus $y = e^x + \frac{x^3}{6} + x + 2$.

9. $\frac{dr}{dq} = 0.7$

$r = \int 0.7dq = 0.7q + C$

If $q = 0$, r must be 0, so $0 = 0 + C$, $C = 0$. Thus $r = 0.7q$. Since $r = pq$, we have

$p = \frac{r}{q} = \frac{0.7q}{q} = 0.7$. The demand function is $p = 0.7$.

10. $\frac{dr}{dq} = 10 - \frac{1}{16}q$

$r = \int \left[10 - \frac{1}{16}q\right]dq = 10q - \frac{1}{32}q^2 + C$

When $q = 0$, then $r = 0$, so $C = 0$ and

$r = 10q - \frac{1}{32}q^2$. Since $r = pq$, then

$p = \frac{r}{q} = 10 - \frac{1}{32}q$. The demand function is

$p = 10 - \frac{1}{32}q$.

11. $\frac{dr}{dq} = 275 - q - 0.3q^2$

Thus $r = \int \left(275 - q - 0.3q^2\right)dq$

$= 275q - 0.5q^2 - 0.1q^3 + C$. When $q = 0$, r must be 0, so $C = 0$ and $r = 275q - 0.5q^2 - 0.1q^3$.

Since $r = pq$, then $p = \frac{r}{q} = 275 - 0.5q - 0.1q^2$.

Thus the demand function is

$p = 275 - 0.5q - 0.1q^2$.

12. $\frac{dr}{dq} = 5000 - 3(2q + 2q^3)$, so

$r = \int (5000 - 6q - 6q^3)dq$

$= 5000q - 3q^2 - \frac{3q^4}{2} + C$

When $q = 0$, then $r = 0$, so $C = 0$ and

$r = 5000q - 3q^2 - \frac{3q^4}{2}$. Since $r = pq$, then

$p = \frac{r}{q} = 5000 - 3q - \frac{3q^3}{2}$. Therefore the demand function is $p = 5000 - 3q - \frac{3q^3}{2}$.

13. $\frac{dc}{dq} = 1.35$

$c = \int 1.35dq = 1.35q + C$

When $q = 0$, then $c = 200$, so $200 = 0 + C$, or $C = 200$. Thus $c = 1.35q + 200$.

14. $\frac{dc}{dq} = 2q + 75$

$c = \int (2q + 75)dq = q^2 + 75q + C$

When $q = 0$, then $c = 2000$, so $C = 2000$. Thus the cost function is $c = q^2 + 75q + 2000$.

15. $\frac{dc}{dq} = 0.08q^2 - 1.6q + 6.5$

$c = \int \left(0.08q^2 - 1.6q + 6.5\right)dq$

$\frac{0.08}{3}q^3 - 0.8q^2 + 6.5q + C$. If $q = 0$, then $c = 8000$, from which $C = 8000$. Hence

$c = \frac{0.08}{3}q^3 - 0.8q^2 + 6.5q + 8000$. If $q = 25$,

substituting gives $c(25) = 8079\frac{1}{6}$ or \$8079.17.

16. $\frac{dc}{dq} = 0.000204q^2 - 0.046q + 6$

$c = \int (0.000204q^2 - 0.046q + 6)dq$

$= 0.000068q^3 - 0.023q^2 + 6q + C$

When $q = 0$, then $c = 15{,}000$, from which $C = 15{,}000$. The cost function is

$c = 0.000068q^3 - 0.023q^2 + 6q + 15{,}000$. When $q = 200$, substitution gives $c(200) = 15{,}824$.

17. $G = \int \left[-\frac{P}{25} + 2\right]dP = -\frac{P^2}{50} + 2P + C$

When $P = 10$, then $G = 38$, so $38 = -2 + 20 + C$, from which $C = 20$. Thus

$G = -\frac{1}{50}P^2 + 2P + 20$.

18. $\frac{dy}{dx} = -1.5 - x$

$y = \int (-1.5 - x)dx = -1.5x - \frac{x^2}{2} + C$

When $x = 1$, then $y = 57.3$, so

$57.3 = -1.5 - 0.5 + C$, or $C = 59.3$. Thus $y = -1.5x - 0.5x^2 + 59.3$.

19. $v = \int -\frac{(P_1 - P_2)r}{2l\eta} dr = -\frac{(P_1 - P_2)r^2}{4l\eta} + C$

Since $v = 0$ when $r = R$, then $0 = -\frac{(P_1 - P_2)R^2}{4l\eta} + C$, so $C = \frac{(P_1 - P_2)R^2}{4l\eta}$. Thus

$v = -\frac{(P_1 - P_2)r^2}{4l\eta} + \frac{(P_1 - P_2)R^2}{4l\eta} = \frac{(P_1 - P_2)(R^2 - r^2)}{4l\eta}$.

20. $\frac{dr}{dq} = 100 - 3q^2$

$r = \int (100 - 3q^2) dq = 100q - q^3 + C$

When $q = 0$, then $r = 0$, so $C = 0$ and $r = 100q - q^3$. Since $r = pq$, then $p = \frac{r}{q} = 100 - q^2$.

$\eta = \frac{\frac{p}{q}}{\frac{dp}{dq}} = \frac{\frac{p}{q}}{-2q} = -\frac{p}{2q^2}$

When $q = 5$, then $p = 75$, so $\eta = \frac{-75}{2(25)} = -\frac{3}{2}$.

21. $\frac{dc}{dq} = 0.003q^2 - 0.4q + 40$

$c = \int (0.003q^2 - 0.4q + 40) dq = 0.001q^3 - 0.2q^2 + 40q + C$

When $q = 0$, then $c = 5000$, so

$5000 = 0 - 0 + 0 + C$, or $C = 5000$. Thus $c = 0.001q^3 - 0.2q^2 + 40q + 5000$. When $q = 100$, then $c = 8000$. Since

Avg. Cost $= \bar{c} = \frac{\text{Total Cost}}{\text{Quantity}} = \frac{c}{q}$, when $q = 100$, we have $\bar{c} = \frac{8000}{100} = \$80$. (Observe that knowing $\frac{dc}{dq} = 27.50$ when $q = 50$ is not relevant to the problem.)

22. $f''(x) = 30x^4 + 12x$

$f'(x) = \int (30x^4 + 12x)dx = 6x^5 + 6x^2 + C_1$

$f'(1) = 10$, so $10 = 6 + 6 + C_1$ and $C_1 = -2$.

$f'(x) = 6x^5 + 6x^2 - 2$

$f(x) = \int (6x^5 + 6x^2 - 2)dx = x^6 + 2x^3 - 2x + C_2$

Thus

$f(965.335245) - f(-965.335245)$

$= [(965.335245)^6 + 2(965.335245)^3 - 2(965.335245) + C_2]$
$\quad - [(-965.335245)^6 + 2(-965.335245)^3 - 2(-965.335245) + C_2]$

$= 3{,}598{,}280{,}000$

Principles in Practice 14.4

1. Using the values given, $\frac{dT}{dt} = -0.5(70 - 60)e^{-0.5t} = -5e^{-0.5t}$

$T(t) = \int \frac{dT}{dt} dt = \int -5e^{-0.5t} dt = 10e^{-0.5t} + C$

2. The number of words memorized is $v(t)$.

$v(t) = \int \frac{35}{t+1} dt = 35 \ln|t + 1| + C$.

Problems 14.4

1. Let $u = x + 5 \Rightarrow du = 1dx = dx$

$\int (x+5)^7 [dx] = \int u^7 du = \frac{u^8}{8} + C = \frac{(x+5)^8}{8} + C$

2. $\int 15(x+2)^4 dx = 15 \int (x+2)^4 [dx] = 15 \cdot \frac{(x+2)^5}{5} + C = 3(x+2)^5 + C$

3. Let $u = x^2 + 3 \Rightarrow du = 2x\, dx$

$\int 2x(x^2 + 3)^5 dx = \int (x^2 + 3)^5 [2x\, dx] = \int u^5 du = \frac{u^6}{6} + C$

$= \frac{(x^2 + 3)^6}{6} + C$

4. Let $u = x^3 + 5x^2 + 6 \Rightarrow du = (3x^2 + 10x)dx$.

$\int (3x^2 + 10x)(x^3 + 5x^2 + 6) dx$

$= \int (x^3 + 5x^2 + 6)^1 [(3x^2 + 10x) dx]$

$= \int u\, du = \frac{u^2}{2} + C$

$= \frac{(x^3 + 5x^2 + 6)^2}{2} + C$

5. Let $u = y^3 + 3y^2 + 1 \Rightarrow du = \left(3y^2 + 6y\right)dy$

$$\int\left(3y^2 + 6y\right)\left(y^3 + 3y^2 + 1\right)^{\frac{2}{3}} dy$$
$$= \int\left(y^3 + 3y^2 + 1\right)^{\frac{2}{3}}\left[\left(3y^2 + 6y\right)dy\right]$$
$$= \int u^{\frac{2}{3}} du = \frac{u^{\frac{5}{3}}}{\frac{5}{3}} + C$$
$$= \frac{3}{5}\left(y^3 + 3y^2 + 1\right)^{\frac{5}{3}} + C$$

6. $\int(15t^2 - 6t + 1)(5t^3 - 3t^2 + t)^{17} dt$
$$= \int(5t^3 - 3t^2 + t)^{17}[(15t^2 - 6t + 1)dt]$$
$$= \frac{(5t^3 - 3t^2 + t)^{18}}{18} + C$$

7. Let $u = 3x - 1 \Rightarrow du = 3\ dx$

$$\int\frac{5}{(3x-1)^3}dx = \frac{5}{3}\int\frac{1}{(3x-1)^3}[3\ dx]$$
$$= \frac{5}{3}\int\frac{1}{u^3}du = \frac{5}{3}\int u^{-3}du$$
$$= \frac{5}{3}\cdot\frac{u^{-2}}{-2} + C = -\frac{5(3x-1)^{-2}}{6} + C$$

8. $\int\frac{4x}{\left(2x^2-7\right)^{10}}dx = \int\left(2x^2-7\right)^{-10}[4x\ dx]$
$$= -\frac{\left(2x^2-7\right)^{-9}}{9} + C$$

9. Let $u = 2x - 1 \Rightarrow du = 2\ dx$.

$$\int\sqrt{2x-1}dx = \int(2x-1)^{\frac{1}{2}}dx$$
$$= \frac{1}{2}\int(2x-1)^{\frac{1}{2}}[2\ dx]$$
$$= \frac{1}{2}\int u^{\frac{1}{2}}du = \frac{1}{2}\cdot\frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{1}{3}(2x-1)^{\frac{3}{2}} + C$$

10. Let $u = x - 5 \Rightarrow du = dx$.

$$\int\frac{1}{\sqrt{x-5}}dx = \int(x-5)^{-\frac{1}{2}}[dx]$$
$$\int u^{-1/2}du = \frac{u^{1/2}}{\frac{1}{2}} + C$$
$$= \frac{(x-5)^{\frac{1}{2}}}{\frac{1}{2}} + C = 2\sqrt{x-5} + C\ .$$

11. Let $u = 7x - 6 \Rightarrow du = 7\ dx$

$$\int(7x-6)^4 dx = \frac{1}{7}\int(7x-6)^4[7\ dx]$$
$$= \frac{1}{7}\int u^4 du = \frac{1}{7}\cdot\frac{u^5}{5} + C$$
$$= \frac{(7x-6)^5}{35} + C$$

12. $\int x^2\left(3x^3+7\right)^3 dx = \frac{1}{9}\int\left(3x^3+7\right)^3\left[9x^2dx\right]$
$$= \frac{1}{9}\cdot\frac{\left(3x^3+7\right)^4}{4} + C$$
$$= \frac{\left(3x^3+7\right)^4}{36} + C$$

13. Let $v = 5u^2 - 9 \Rightarrow dv = 10u\,du$

$$\int u(5u^2-9)^{14}du = \frac{1}{10}\int(5u^2-9)^{14}[10u\,du]$$
$$\frac{1}{10}\int v^{14}dv = \frac{1}{10}\cdot\frac{v^{15}}{15} + C = \frac{(5u^2-9)^{15}}{150} + C\ .$$

14. $\int 9x\sqrt{1+2x^2}dx = \frac{9}{4}\int\left(1+2x^2\right)^{\frac{1}{2}}[4x\ dx]$
$$= \frac{9}{4}\cdot\frac{\left(1+2x^2\right)^{\frac{3}{2}}}{\frac{3}{2}} + C$$
$$= \frac{3\left(1+2x^2\right)^{\frac{3}{2}}}{2} + C$$

15. Let $u = 27 + x^5 \Rightarrow du = 5x^4dx$

$$\int 4x^4\left(27+x^5\right)^{\frac{1}{3}}dx = \frac{4}{5}\int\left(27+x^5\right)^{\frac{1}{3}}\left[5x^4dx\right]$$
$$= \frac{4}{5}\int u^{\frac{1}{3}}du = \frac{4}{5}\cdot\frac{u^{\frac{4}{3}}}{\frac{4}{3}} + C$$
$$= \frac{3}{5}\left(27+x^5\right)^{\frac{4}{3}} + C$$

16. Let $u = 4 - 5x \Rightarrow du = -5dx$.

$$\int(4-5x)^9dx = -\frac{1}{5}\int(4-5x)^9[-5\ dx]$$
$$= -\frac{1}{5}\int u^9du = -\frac{1}{5}\cdot\frac{u^{10}}{10} + C = -\frac{1}{50}(4-5x)^{10} + C$$

17. Let $u = 3x \Rightarrow du = 3\ dx$

$$\int 3e^{3x}dx = \int e^{3x}[3\ dx]$$
$$= \int e^u du = e^u + C = e^{3x} + C$$

18. $\int 5e^{3t+7}dt = \frac{5}{3}\int e^{3t+7}[3\,dt] = \frac{5}{3}e^{3t+7} + C$

19. Let $u = t^2 + t \Rightarrow du = (2t+1)dt$

$$\int(2t+1)e^{t^2+t}dt = \int e^{t^2+t}[(2t+1)\ dt]$$
$$= \int e^u du = e^u + C = e^{t^2+t} + C$$

20. $\int -3w^2e^{-w^3}dw = \int e^{-w^3}\left[-3w^2dw\right] = e^{-w^3} + C$

21. Let $u = 7x^2 \Rightarrow du = 14x\,dx$

$$\int xe^{7x^2}dx = \frac{1}{14}\int e^{7x^2}[14x\ dx] = \frac{1}{14}\int e^u du$$
$$= \frac{1}{14}e^u + C = \frac{1}{14}e^{7x^2} + C$$

22. $\int x^3e^{4x^4}dx = \frac{1}{16}\int e^{4x^4}\left[16x^3dx\right]$
$$= \frac{1}{16}\cdot e^{4x^4} + C = \frac{e^{4x^4}}{16} + C$$

23. Let $u = -3x \Rightarrow du = -3dx$.

$$\int 4e^{-3x}dx = -\frac{4}{3}\int e^{-3x}[-3\ dx]$$
$$= -\frac{4}{3}\int e^u du = -\frac{4}{3}e^u + C = -\frac{4}{3}e^{-3x} + C$$

24. $\int x^4e^{-6x^5}dx = -\frac{1}{30}\int e^{-6x^5}\left[-30x^4dx\right]$
$$= -\frac{1}{30}e^{-6x^5} + C$$

25. Let $u = x + 5 \Rightarrow du = dx$

$$\int\frac{1}{x+5}[dx] = \int\frac{1}{u}du = \ln|u| + C = \ln|x+5| + C$$

26. $\int\frac{12x^2+4x+2}{x+x^2+2x^3}dx$
$$= \int\frac{2}{x+x^2+2x^3}[(1+2x+6x^2)dx]$$
$$= 2\ln\left|x+x^2+2x^3\right| + C$$
$$= \ln[(x+x^2+2x^3)^2] + C$$

27. Let $u = x^3 + x^4 \Rightarrow du = (3x^2+4x^3)dx$

$$\int\frac{3x^2+4x^3}{x^3+x^4}dx = \int\frac{1}{x^3+x^4}\left[\left(3x^2+4x^3\right)dx\right]$$
$$= \int\frac{1}{u}du = \ln|u| + C$$
$$= \ln\left|x^3+x^4\right| + C$$

28. Let $u = 1 - 3x^2 + 2x^3 \Rightarrow du = (-6x+6x^2)dx$.

$$\int\frac{6x^2-6x}{1-3x^2+2x^3}dx$$
$$= \int\frac{1}{1-3x^2+2x^3}[(-6x+6x^2)dx$$
$$= \int\frac{1}{u}du = \ln|u| + C = \ln\left|1-3x^2+2x^3\right| + C$$

29. Let $u = z^2 - 6 \Rightarrow du = 2z\,dz$

$$\int\frac{6z}{(z^2-6)^5} = 3\int(z^2-6)^{-5}[2z\,dz]$$
$$= 3\int u^{-5}du = 3\frac{u^{-4}}{-4} + C = -\frac{3}{4}(z^2-6)^{-4} + C$$

30. $\int\frac{3}{(5v-1)^4}dv = \frac{3}{5}\int(5v-1)^{-4}[5\,dv]$
$$= \frac{3}{5}\cdot\frac{(5v-1)^{-3}}{-3} + C$$
$$= -\frac{1}{5}(5v-1)^{-3} + C$$

31. $\int \frac{4}{x}dx = 4\int \frac{1}{x}dx = 4\ln|x| + C$

32. $\int \frac{3}{1+2y}dy = 3\cdot\frac{1}{2}\int\frac{1}{1+2y}[2\,dy]$
$= \frac{3}{2}\ln|1+2y|+C$

33. Let $u=s^3+5 \Rightarrow du = 3s^2\,ds$
$\int \frac{s^2}{s^3+5}ds = \frac{1}{3}\int\frac{1}{s^3+5}\left[3s^2ds\right]$
$= \frac{1}{3}\int\frac{1}{u}du = \frac{1}{3}\ln|u|+C = \frac{1}{3}\ln\left|s^3+5\right|+C$

34. $\int\frac{2x^2}{3-4x^3}dx = 2\left(-\frac{1}{12}\right)\int\frac{1}{3-4x^3}\left[-12x^2dx\right]$
$= -\frac{1}{6}\ln\left|3-4x^3\right|+C$

35. Let $u=4-2x \Rightarrow du=-2\,dx$
$\int\frac{5}{4-2x}dx = -\frac{5}{2}\int\frac{1}{4-2x}[-2\,dx]$
$= -\frac{5}{2}\int\frac{1}{u}du = -\frac{5}{2}\ln|u|+C = -\frac{5}{2}\ln|4-2x|+C$

36. $\int\frac{7t}{5t^2-6}dt = 7\cdot\frac{1}{10}\int\frac{1}{5t^2-6}[10t\,dt]$
$= \frac{7}{10}\ln\left|5t^2-6\right|+C$

37. $\int\sqrt{5x}\,dx = \sqrt{5}\int x^{1/2}dx = \sqrt{5}\,\frac{x^{\frac{3}{2}}}{\frac{3}{2}}+C$
$= \frac{2\sqrt{5}}{3}x^{\frac{3}{2}}+C$

38. $\int\frac{1}{(3x)^6}dx = \frac{1}{3}\int(3x)^{-6}[3\,dx]$
$= \frac{1}{3}\cdot\frac{(3x)^{-5}}{-5}+C$
$= -\frac{1}{15}(3x)^{-5}+C$

39. Let $u=x^2-4 \Rightarrow du=2x\,dx$
$\int\frac{x}{\sqrt{x^2-4}}dx = \frac{1}{2}\int\left(x^2-4\right)^{-\frac{1}{2}}[2x\,dx]$
$= \frac{1}{2}\int u^{-\frac{1}{2}}du = \frac{1}{2}\cdot\frac{u^{\frac{1}{2}}}{\frac{1}{2}}+C$
$= \sqrt{x^2-4}+C$

40. Let $u = 1-3x \Rightarrow du = -3\,dx.$
$\int\frac{9}{1-3x}dx = -3\int\frac{1}{1-3x}[-3\,dx]$
$= -3\int\frac{1}{u}du = -3\ln|u|+C = -3\ln|1-3x|+C$

41. Let $u=y^4+1 \Rightarrow du=4y^3dy$
$\int 2y^3e^{y^4+1}dy = 2\int y^3e^{y^4+1}dy$
$= 2\cdot\frac{1}{4}\int e^{y^4+1}\left[4y^3dy\right]$
$= \frac{1}{2}\int e^u du = \frac{1}{2}e^u+C$
$= \frac{1}{2}e^{y^4+1}+C$

42. $\int 2\sqrt{2x-1}\,dx = \int(2x-1)^{\frac{1}{2}}[2\,dx]$
$= \frac{(2x-1)^{\frac{3}{2}}}{\frac{3}{2}}+C$
$= \frac{2}{3}(2x-1)^{\frac{3}{2}}+C$

43. Let $u=-2v^3+1 \Rightarrow du=-6v^2dv$
$\int v^2e^{-2v^3+1}dv = -\frac{1}{6}\int e^{-2v^3+1}\left[-6v^2dv\right]$
$= -\frac{1}{6}\int e^u du = -\frac{1}{6}e^u+C$
$= -\frac{1}{6}e^{-2v^3+1}+C$

44. $\int\frac{x^2}{\sqrt[3]{2x^3+9}}dx = \frac{1}{6}\int\left(2x^3+9\right)^{-\frac{1}{3}}\left[6x^2dx\right]$
$= \frac{1}{6}\cdot\frac{\left(2x^3+9\right)^{\frac{2}{3}}}{\frac{2}{3}}+C$
$= \frac{1}{4}\left(2x^3+9\right)^{\frac{2}{3}}+C$

45. $\int\left(e^{-5x}+2e^x\right)dx = \int e^{-5x}dx+2\int e^x dx$
$= -\frac{1}{5}\int e^{-5x}[-5\,dx]+2\int e^x dx$
$= -\frac{1}{5}e^{-5x}+2e^x+C$

46. $\int 4\sqrt[3]{y+1}\,dy = 4\int(y+1)^{\frac{1}{3}}[dy]$
$= 4\cdot\frac{(y+1)^{\frac{4}{3}}}{\frac{4}{3}}+C = 3(y+1)^{\frac{4}{3}}+C$

47. $\int(8x+10)(7-2x^2-5x)^3dx$
$= -2\int(7-2x^2-5x)^3[(-4x-5)dx]$
$= -2\cdot\frac{(7-2x^2-5x)^4}{4}+C$
$= -\frac{1}{2}(7-2x^2-5x)^4+C$

48. $\int 2ye^{3y^2}dy = 2\cdot\frac{1}{6}\int e^{3y^2}[6y\,dy] = \frac{1}{3}e^{3y^2}+C$

49. $\int\frac{x^2+2}{x^3+6x}dx = \frac{1}{3}\int\frac{1}{x^3+6x}\left[\left(3x^2+6\right)dx\right]$
$= \frac{1}{3}\ln\left|x^3+6x\right|+C$

50. $\int(e^x+2e^{-3x}-e^{5x})dx$
$= \int e^x dx-\frac{2}{3}\int e^{-3x}[(-3)dx]-\frac{1}{5}\int e^{5x}[5\,dx]$
$= e^x-\frac{2}{3}e^{-3x}-\frac{1}{5}e^{5x}+C$

51. $\int\frac{16s-4}{3-2s+4s^2}dx = 2\int\frac{1}{3-2s+4s^2}[(8s-2)ds]$
$= 2\ln\left|3-2s+4s^2\right|+C$

52. $\int(6t^2+4t)(t^3+t^2+1)^6dt$
$= 2\int(t^3+t^2+1)^6[(3t^2+2t)dt]$
$= 2\cdot\frac{(t^3+t^2+1)^7}{7}+C$
$= \frac{2}{7}(t^3+t^2+1)^7+C$

53. $\int x\left(2x^2+1\right)^{-1}dx = \int\frac{x}{2x^2+1}dx$
$= \frac{1}{4}\int\frac{1}{2x^2+1}[4x\,dx]$
$= \frac{1}{4}\ln\left(2x^2+1\right)+C$

54. $\int(8w^5+w^2-2)(6w-w^3-4w^6)^{-4}dw$
$= -\frac{1}{3}\int(6w-w^3-4w^6)^{-4}[(6-3w^2-24w^5)dw]$
$= -\frac{1}{3}\cdot\frac{(6w-w^3-4w^6)^{-3}}{-3}+C$
$= \frac{1}{9}(6w-w^3-4w^6)^{-3}+C$

55. $\int-\left(x^2-2x^5\right)\left(x^3-x^6\right)^{-10}dx$
$= -\frac{1}{3}\int\left(x^3-x^6\right)^{-10}\left[\left(3x^2-6x^5\right)dx\right]$
$= -\frac{1}{3}\cdot\frac{\left(x^3-x^6\right)^{-9}}{-9}+C = \frac{1}{27}\left(x^3-x^6\right)^{-9}+C$

56. $\int\frac{3}{5}(v-2)e^{2-4v+v^2}dv$
$= \frac{3}{5}\cdot\frac{1}{2}\int e^{2-4v+v^2}[(2v-4)\,dv]$
$= \frac{3}{10}e^{2-4v+v^2}+C$

57. $\int\left(2x^3+x\right)\left(x^4+x^2\right)dx$
$= \frac{1}{2}\int\left(x^4+x^2\right)^1\left[\left(4x^3+2x\right)dx\right]$
$= \frac{1}{2}\cdot\frac{\left(x^4+x^2\right)^2}{2}+C = \frac{1}{4}\left(x^4+x^2\right)^2+C$

58. $\int\left(e^{3.1}\right)^2 dx = \int e^{6.2} dx = e^{6.2}x + C$, because $e^{6.2}$ is a constant.

59. $\int \frac{7+14x}{(4-x-x^2)^5}dx$
$= -7\int (4-x-x^2)^{-5}[(-1-2x)dx]$
$= -7\frac{(4-x-x^2)^{-4}}{-4} + C$
$= \frac{7}{4}(4-x-x^2)^{-4} + C$

60. $\int\left(e^x - e^{-x}\right)^2 dx = \int\left(e^{2x} - 2 + e^{-2x}\right) dx$
$= \frac{1}{2}\int e^{2x}[2\ dx] - \int 2\ dx + \left(-\frac{1}{2}\right)\int e^{-2x}[-2\ dx]$
$= \frac{1}{2}e^{2x} - 2x - \frac{1}{2}e^{-2x} + C$
$= \frac{1}{2}\left(e^{2x} - e^{-2x}\right) - 2x + C$

61. $u = 4x^3 + 3x^2 - 4$
$du = \left(12x^2 + 6x\right)dx = 6x(2x+1)dx$
$\int x(2x+1)e^{4x^3+3x^2-4}dx$
$= \frac{1}{6}\int e^{4x^3+3x^2-4}[6x(2x+1)dx]$
$= \frac{1}{6}\int e^u du = \frac{1}{6}e^u + C = \frac{1}{6}e^{4x^3+3x^2-4} + C$

62. $\int(u^3 - ue^{6-3u^2})du = \frac{u^4}{4} + \frac{1}{6}\int e^{6-3u^2}[-6u\ du]$
$= \frac{1}{4}u^4 + \frac{1}{6}e^{6-3u^2} + C$

63. $\int x\sqrt{\left(8-5x^2\right)^3}dx = -\frac{1}{10}\int\left(8-5x^2\right)^{\frac{3}{2}}[-10x\ dx]$
$= -\frac{1}{10}\cdot\frac{\left(8-5x^2\right)^{\frac{5}{2}}}{\frac{5}{2}} + C = -\frac{1}{25}\left(8-5x^2\right)^{\frac{5}{2}} + C$

64. $\int e^{-\frac{x}{7}}dx = -7\int e^{-\frac{x}{7}}\left[-\frac{1}{7}dx\right] = -7e^{-\frac{x}{7}} + C$

65. $\int\left(\sqrt{2x} - \frac{1}{\sqrt{2x}}\right)dx = \int\sqrt{2x}dx - \int\frac{1}{\sqrt{2x}}dx$
$= \frac{1}{2}\int(2x)^{\frac{1}{2}}[2\ dx] - \frac{1}{2}\int(2x)^{-\frac{1}{2}}[2\ dx]$
$= \frac{1}{2}\cdot\frac{(2x)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{1}{2}\cdot\frac{(2x)^{\frac{1}{2}}}{\frac{1}{2}} + C = \frac{(2x)^{\frac{3}{2}}}{3} - \sqrt{2x} + C$
$= \frac{2\sqrt{2}}{3}x^{\frac{3}{2}} - \sqrt{2}x^{\frac{1}{2}} + C$

66. $\int 3\frac{x^4}{e^{x^5}}dx = 3\int x^4e^{-x^5}dx = -\frac{3}{5}\int e^{-x^5}[-5x^4\ dx]$
$= -\frac{3}{5}e^{-x^5} + C$

67. $\int\left(x^2+1\right)^2 dx = \int\left(x^4 + 2x^2 + 1\right)dx$
$= \frac{x^5}{5} + \frac{2x^3}{3} + x + C$

68. $\int\left[x\left(x^2-16\right)^2 - \frac{1}{2x+5}\right]dx$
$= \frac{1}{2}\int\left(x^2-16\right)^2[2x\ dx] - \frac{1}{2}\int\frac{1}{2x+5}[2\ dx]$
$= \frac{1}{2}\cdot\frac{\left(x^2-16\right)^3}{3} - \frac{1}{2}\ln|2x+5| + C$
$= \frac{1}{6}\left(x^2-16\right)^3 - \frac{1}{2}\ln|2x+5| + C$

69. $\int\left[\frac{x}{x^2+1} + \frac{x^5}{(x^6+1)^2}\right]dx$
$= \int\frac{x}{x^2+1}dx + \int\frac{x^5}{\left(x^6+1\right)^2}dx$
$= \frac{1}{2}\int\frac{1}{x^2+1}[2x\ dx] + \frac{1}{6}\int\left(x^6+1\right)^{-2}\left[6x^5dx\right]$
$= \frac{1}{2}\ln\left(x^2+1\right) + \frac{1}{6}\cdot\frac{\left(x^6+1\right)^{-1}}{-1} + C$
$= \frac{1}{2}\ln\left(x^2+1\right) - \frac{1}{6\left(x^6+1\right)} + C$

70. $\int\left[\frac{3}{x-1} + \frac{1}{(x-1)^2}\right]dx = \int\frac{3}{x-1}[dx] + \int(x-1)^{-2}[dx]$
$= 3\ln|x-1| + \frac{(x-1)^{-1}}{-1} + C = 3\ln|x-1| - \frac{1}{x-1} + C$

71. $\int\left[\frac{2}{4x+1} - (4x^2-8x^5)(x^3-x^6)^{-8}\right]dx$
$= \frac{1}{2}\int\frac{1}{4x+1}[4\,dx] - \frac{4}{3}\int(x^3-x^6)^{-8}[(3x^2-6x^5)dx]$
$= \frac{1}{2}\ln|4x+1| - \frac{4}{3}\cdot\frac{(x^3-x^6)^{-7}}{-7} + C$
$= \frac{1}{2}\ln|4x+1| + \frac{4}{21}(x^3-x^6)^{-7} + C$

72. $\int\left(r^3+5\right)^2 dr = \int\left(r^6 + 10r^3 + 25\right)dr = \frac{1}{7}r^7 + \frac{5}{2}r^4 + 25r + C$

73. $\int\left[\sqrt{3x+1} - \frac{x}{x^2+3}\right]dx = \int(3x+1)^{\frac{1}{2}}dx - \int\frac{x}{x^2+3}dx = \frac{1}{3}\int(3x+1)^{\frac{1}{2}}[3\ dx] - \frac{1}{2}\int\frac{1}{x^2+3}[2x\ dx]$
$= \frac{1}{3}\cdot\frac{(3x+1)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{1}{2}\ln\left(x^2+3\right) + C = \frac{2}{9}(3x+1)^{\frac{3}{2}} - \ln\sqrt{x^2+3} + C$

74. $\int\left[\frac{x}{3x^2+5} - \frac{x^2}{(x^3+1)^3}\right]dx = \frac{1}{6}\int\frac{1}{3x^2+5}[6x\,dx] - \frac{1}{3}\int(x^3+1)^{-3}[3x^2\ dx]$
$= \frac{1}{6}\ln\left|3x^2+5\right| - \frac{1}{3}\cdot\frac{(x^3+1)^{-2}}{-2} + C = \frac{1}{6}\ln\left|3x^2+5\right| + \frac{1}{6}(x^3+1)^{-2} + C$

75. Let $u = \sqrt{x} \Rightarrow du = \frac{1}{2}x^{-\frac{1}{2}}dx = \frac{1}{2\sqrt{x}}dx$.
$\int\frac{e^{\sqrt{x}}}{\sqrt{x}}dx = 2\int e^{\sqrt{x}}\left[\frac{1}{2\sqrt{x}}dx\right]$
$= 2\int e^u du = 2e^u + C = 2e^{\sqrt{x}} + C$

76. $\int\left(e^5 - 3^e\right)dx = \left(e^5 - 3^e\right)x + C$, because $e^5 - 3^e$ is a constant.

77. $\int\frac{1+e^{2x}}{4e^x}dx = \frac{1}{4}\int\left(\frac{1}{e^x} + \frac{e^{2x}}{e^x}\right)dx$
$= \frac{1}{4}\int\left(e^{-x} + e^x\right)dx$
$= -\frac{1}{4}\int e^{-x}[-1\ dx] + \frac{1}{4}\int e^x dx$
$= -\frac{1}{4}e^{-x} + \frac{1}{4}e^x + C$

78. $\int \frac{2}{t^2}\sqrt{\frac{1}{t}+9}\,dt = -2\int\left(\frac{1}{t}+9\right)^{\frac{1}{2}}\left[-\frac{1}{t^2}dt\right]$

$= -2\frac{\left(\frac{1}{t}+9\right)^{\frac{3}{2}}}{\frac{3}{2}}+C$

$= -\frac{4}{3}\left(\frac{1}{t}+9\right)^{\frac{3}{2}}+C$

79. Let $u=\ln\left(x^2+2x\right) \Rightarrow du=\frac{1}{x^2+2x}(2x+2)dx$

$\int\frac{x+1}{x^2+2x}\ln\left(x^2+2x\right)dx$

$=\frac{1}{2}\int\ln\left(x^2+2x\right)\left[\frac{2x+2}{x^2+2x}dx\right]$

$=\frac{1}{2}\int u\,du=\frac{1}{2}\cdot\frac{u^2}{2}+C=\frac{1}{4}\ln^2\left(x^2+2x\right)+C$

80. Let $u=\sqrt[3]{8x^4}=2x^{\frac{4}{3}}\Rightarrow du=\frac{8}{3}x^{\frac{1}{3}}dx$

$\int\sqrt[3]{x}e^{\sqrt[3]{8x^4}}dx=\frac{3}{8}\int e^{2x^{\frac{4}{3}}}\left[\frac{8}{3}x^{\frac{1}{3}}dx\right]=\frac{3}{8}\int e^u du$

$=\frac{3}{8}e^u+C=\frac{3}{8}e^{\sqrt[3]{8x^4}}+C$

81. $y=\int(3-2x)^2dx=-\frac{1}{2}\int(3-2x)^2[-2\,dx]$

$=-\frac{1}{2}\cdot\frac{(3-2x)^3}{3}+C=-\frac{1}{6}(3-2x)^3+C$

$y(0)=1$ implies $1=-\frac{1}{6}(27)+C$, so $C=\frac{11}{2}$.

Thus $y=-\frac{1}{6}(3-2x)^3+\frac{11}{2}$.

82. $y=\frac{1}{2}\int\frac{1}{x^2+6}[2x\,dx]=\frac{1}{2}\ln\left(x^2+6\right)+C$

$y(1)=0$ implies $0=\frac{1}{2}\ln(7)+C$, so $C=-\frac{1}{2}\ln 7$.

Thus $y=\frac{1}{2}\left[\ln\left(x^2+6\right)-\ln 7\right]$, or

$y=\ln\sqrt{\frac{x^2+6}{7}}$

83. $y''=\frac{1}{x^2}$

$y'=\int x^{-2}dx=-x^{-1}+C_1$

$y'(-2)=3$ implies $3=\frac{1}{2}+C_1$, so $C_1=\frac{5}{2}$. Thus

$y'=-x^{-1}+\frac{5}{2}$.

$y=\int\left(-x^{-1}+\frac{5}{2}\right)dx=-\int\frac{1}{x}dx+\int\frac{5}{2}dx$

$=-\ln|x|+\frac{5}{2}x+C_2$

$y(1)=2$ implies that $2=0+\frac{5}{2}+C_2$, so

$C_2=-\frac{1}{2}$. Thus

$y=-\ln|x|+\frac{5}{2}x-\frac{1}{2}=\ln\left|\frac{1}{x}\right|+\frac{5}{2}x-\frac{1}{2}$.

84. $y''=(x+1)^{3/2}$

$y'=\int(x+1)^{\frac{3}{2}}dx=\frac{2}{5}(x+1)^{\frac{5}{2}}+C_1$

$y'(3)=0\Rightarrow 0=\frac{2}{5}\cdot 32+C_1\Rightarrow C_1=-\frac{64}{5}$, so

$y'=\frac{2}{5}(x+1)^{\frac{5}{2}}-\frac{64}{5}$

$y=\int\left[\frac{2}{5}(x+1)^{\frac{5}{2}}-\frac{64}{5}\right]dx$

$=\frac{2}{5}\cdot\frac{(x+1)^{\frac{7}{2}}}{\frac{7}{2}}-\frac{64}{5}x+C_2$

$=\frac{4}{35}(x+1)^{\frac{7}{2}}-\frac{64}{5}x+C_2$

$y(3)=0$ implies $0=\frac{4}{35}\cdot 128-\frac{64}{5}(3)+C_2$, so

$C_2=\frac{832}{35}$. Thus $y=\frac{4}{35}(x+1)^{\frac{7}{2}}-\frac{64}{5}x+\frac{832}{35}$.

85. $V(t)=\int\frac{dV}{dt}dt=\int 8e^{0.05t}dt$

$=\frac{8}{0.05}\int e^{0.05t}[0.05\,dt]$

$=160e^{0.05t}+C$

The house cost $350,000 to build, so $V(0)=350$.

$350=160e^0+C=160+C$

$190=C$

$V(t)=160e^{0.05t}+190$

86. $I(t)=\int\frac{dI}{dt}dt=\int\frac{12}{2t+50}dt$

$=6\ln|2t+50|+C$

Since the expected life span was 63 years in 1940, $I(0)=63$.

$63=6\ln|50|+C$

$C=63-6\ln 50\approx 39.53$

$I(t)=6\ln|2t+50|+39.53$

$I(58)=6\ln|166|+39.53\approx 70.20$

The expected life span for people born in 1998 (58 years after 1940) is about 70 years.

87. Note that $r>0$.

$C=\int\left[\frac{Rr}{2K}+\frac{B_1}{r}\right]dr=\int\frac{Rr}{2K}dr+\int\frac{B_1}{r}dr$

$=\frac{R}{2K}\int r\,dr+B_1\int\frac{1}{r}dr$

$=\frac{R}{2K}\cdot\frac{r^2}{2}+B_1\ln|r|+B_2$

Thus we obtain $C=\frac{Rr^2}{4K}+B_1\ln|r|+B_2$.

88. $f(x)=\int(e^{3x+2}-3x)dx=\frac{1}{3}e^{3x+2}-\frac{3}{2}x^2+C$

$f\left(\frac{1}{3}\right)=2$ implies $2=\frac{1}{3}e^3-\frac{1}{6}+C$, so

$C=\frac{13}{6}-\frac{1}{3}e^3$. Thus,

$f(x)=\frac{1}{3}e^{3x+2}-\frac{3}{2}x^2+\frac{13}{6}-\frac{1}{3}e^3$,

$f(2)=\frac{1}{3}e^8-6+\frac{13}{6}-\frac{1}{3}e^3$

$=\frac{1}{6}(2e^8-2e^3-23)\approx 983.12$

Problems 14.5

1. $\int\frac{2x^6+8x^4-4x}{2x^2}dx$

$=\int\left(\frac{2x^6}{2x^2}+\frac{8x^4}{2x^2}-\frac{4x}{2x^2}\right)dx$

$=\int x^4\,dx+4\int x^2\,dx-2\int\frac{1}{x}dx$

$=\frac{x^5}{5}+\frac{4}{3}x^3-2\ln|x|+C$

2. $\int\frac{9x^2+5}{3x}dx=\int\left(3x+\frac{5}{3x}\right)dx$

$=\frac{3}{2}x^2+\frac{5}{3}\ln|x|+C$

3. $\int\left(3x^2+2\right)\sqrt{2x^3+4x+1}\,dx$

$=\frac{1}{2}\int\left(2x^3+4x+1\right)^{\frac{1}{2}}\left[\left(6x^2+4\right)dx\right]$

$=\frac{1}{2}\cdot\frac{\left(2x^3+4x+1\right)^{\frac{3}{2}}}{\frac{3}{2}}+C$

$=\frac{1}{3}\left(2x^3+4x+1\right)^{\frac{3}{2}}+C$

4. $\int\frac{x}{\sqrt[4]{x^2+1}}dx=\frac{1}{2}\int\left(x^2+1\right)^{-\frac{1}{4}}[2x\,dx]$

$=\frac{1}{2}\cdot\frac{\left(x^2+1\right)^{\frac{3}{4}}}{\frac{3}{4}}+C$

$=\frac{2}{3}\left(x^2+1\right)^{\frac{3}{4}}+C$

5. $\int\frac{9}{\sqrt{2-3x}}dx=9\int(2-3x)^{-1/2}dx$

$=9\left(-\frac{1}{3}\right)\int(2-3x)^{-1/2}[-3\,dx]$

$=-3\frac{(2-3x)^{1/2}}{\frac{1}{2}}+C=-6\sqrt{2-3x}+C$

6. $\int\frac{2xe^{x^2}}{e^{x^2}-2}dx=\int\frac{1}{e^{x^2}-2}\left[2xe^{x^2}dx\right]$

$=\ln\left|e^{x^2}-2\right|+C$