Math 120 Quiz #7
Nov. 29, 2010

Problem 1

Use the properties of logarithms to write the following expression in terms of $\log x$, $\log y$ and $\log z$:

$$\log\left(\frac{1}{x^{1/3}}\left(\frac{y^4}{x^2z^3}\right)^{1/5}\right)$$

Solution

The main properties of logs that you need are $\log ab = \log a + \log b$ and $\log a^b = b \log a$.

$$\begin{aligned}
\log\left(\frac{1}{x^{1/3}}\left(\frac{y^4}{x^2z^3}\right)^{1/5}\right) &= \log\left(\frac{1}{x^{1/3}}\right) + \log\left(\left(\frac{y^4}{x^2z^3}\right)^{1/5}\right) \\
&= -\log\left(x^{1/3}\right) + \frac{1}{5}\log\left(\frac{y^4}{x^2z^3}\right) \\
&= -\frac{1}{3}\log x + \frac{1}{5}(\log(y^4) - \log(x^2) - \log(z^3)) \\
&= -\frac{1}{3}\log x + \frac{1}{5}(4\log y - 2\log x - 3\log z) \\
&= -\frac{1}{3}\log x + \frac{4}{5}\log y - \frac{2}{5}\log x - \frac{3}{5}\log z \\
&= -\frac{11}{15}\log x + \frac{4}{5}\log y - \frac{3}{5}\log z
\end{aligned}$$

Problem 2

I woke up this morning and decided that I would cease being a kind and understanding professor and begin docking students points for late homeworks. Assume that homeworks turned in on time (at $t = 0$) are worth 100 and that homeworks turned in 1 day late are worth only 90 points.

Furthermore, assume that the grade of a homework decays according to the equation

$$G(t) = G_0 e^{\lambda t}.$$

Problem 2.a

Determine the value of G_0 and λ.

Solution

Plug in $G(0) = 100$ to determine G_0:

$$\begin{aligned} 100 &= G_0 e^{\lambda(0)} \\ 100 &= G_0 \end{aligned}$$

Now use that value along with $G(1) = 90$ to determine λ.

$$\begin{aligned} 90 &= 100 e^{\lambda(1)} \\ \frac{9}{10} &= e^{\lambda} \\ \log\left(\frac{9}{10}\right) &= \lambda \end{aligned}$$

This leaves us with the function

$$\begin{aligned} G(t) &= 100 e^{\log\left(\frac{9}{10}\right)t} \\ G(t) &= 100\left(\frac{9}{10}\right)^t \end{aligned}$$

Problem 2.b

Using the values of G_0 and λ, determine how many days must pass for a homework to be worth 50 points.

Solution

We are trying to determine what t value satisfies the equation $G(t) = 50$. All that is needed is to solve that equation

$$\begin{aligned} 50 &= 100\left(\frac{9}{10}\right)^t \\ \frac{1}{2} &= \left(\frac{9}{10}\right)^t \\ \log\left(\frac{1}{2}\right) &= t\log\left(\frac{9}{10}\right) \\ t &= \frac{\log 1/2}{\log 9/10} \end{aligned}$$