Math 120 Quiz #6 Nov. 15, 2010

Problem 1

Solve the linear system

$$2x - 3y + 1 = 0$$
$$x + y - 7 = 0$$

Solution

I'll just use substitution, but you could also have used elimination. Solving the second equation for \boldsymbol{y} gives

$$y = 7 - x$$

Plugging in that value into that first equation

$$2x - 3(7 - x) + 1 = 0$$

$$2x - 21 + 3x + 1 = 0$$

$$5x - 20 = 0$$

$$x = 4$$

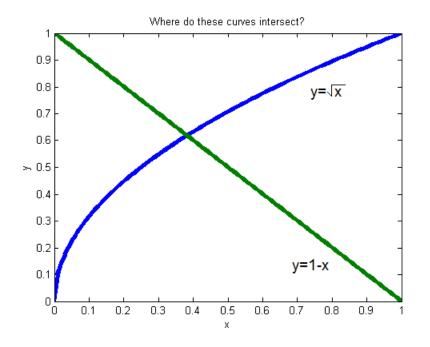
Now that we have x we can plug in to the second equation above

$$4 + y - 7 = 0$$
$$y = 3$$

So the solution is (x, y) = (4, 3).

Problem 2

Examine the graph below:



Determine where these two curves, $y = \sqrt{x}$ and y = 1 - x, intersect.

Solution

To find the intersection of these two curves, we need to determine at what y value they are equal. Setting the two equations equal gives

$$\sqrt{x} = 1 - x$$
$$x = (1 - x)^{2}$$
$$x = 1 - 2x + x^{2}$$
$$x^{2} - 3x + 1 = 0$$

Now we need only use the quadratic formula to solve that equation

$$x = \frac{3 \pm \sqrt{(-3)^2 - 4(1)(1)}}{2(1)}$$
$$x = \frac{1}{2}(3 \pm \sqrt{5})$$

This is troubling because it gives two solutions when in the picture above we have only one solution. To determine which solution is appropriate, we will see which of them is less than 1, since the point of intersection is clearly less than 1.

$$x_1 = \frac{1}{2}(3 + \sqrt{5}) \qquad \qquad = \frac{3}{2} + \frac{1}{2}\sqrt{5} > 1$$

That can't be the solution, so the solution must be $x = \frac{1}{2}(3 - \sqrt{5})$.