Introduction

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One of the fascinating features of mathematics is that the same ideas, concepts, techniques, models, and structures find applications in diverse disciplines. Or that results developed in one application find unexpected applications elsewhere. Within mathematics itself one finds a similar phenomenon: the same structure occurs with different guises in various mathematical problem areas.

This book focuses on these aspects of some discrete structures. In the past sixty years, these structures have appeared in such diverse areas as consensus theory, voting theory, optimization, location theory, clustering, classification, representation, and other areas of discrete mathematics. The ideas, techniques and concepts discussed here have found applications in different disciplines as Biology, Psychology, Economics, Operations Research, Social Choice, Physics, and Chemistry.

A whole series of books could have been written on the topics in book. The aim of a single volume necessarily has to be quite modest. What we would like to achieve is to raise the interest of the reader for the discrete structures and techniques discussed here and also for the many applications and future possibilities of the ideas presented here. We hope that graduate students and researchers from one area will get acquainted with other areas and be able to use these new ideas towards a interdisciplinary approach to Discrete Applied Mathematics.

The design of the book is rather loose: we have asked twelve authors to contribute a chapter on a topic of their own choice within the areas described above. This has resulted in ten chapters that cover a wide range, from applications in biology, economics, and logistics to discussions of areas in discrete mathematics such as centrality in trees, intersection graphs, and median graphs, from surveys on specific topics such as 2×2 tables, the majority rule, and location functions to discussions of future possibilities, but also new results such as generalized centrality in trees and axiomatic characterization of the median function on cube-free networks. The idea of consensus in various forms reappears in several chapters, sometimes in the guise of voting procedures or location problems. On closer inspection the common features may become more apparent.

Another way by which the connections of the chapters in this book are defined is that all authors have collaborated more or less closely with F.R. 'Buck' McMorris. This may have been as co-author, as member of the same PhD-committee, or as having a major common interest in some problems or theories. When we look at the topics in the chapters of this book, then these are precisely the ones to which McMorris has dedicated his mathematical career. The idea for this book arose at the occasion of his retirement as dean of the College of Science and Letters at the Illinois Institute of Technology, IIT, in Chicago. To highlight this unifying idea, a chapter is added at the end of the book discussing the contributions of McMorris to discrete mathematics and its applications.

In the remaining part of the introduction, we will give a short overview of the individual chapters so as to help orient the reader in the technical landscape of the research topics covered therein.

A phylogenetic tree in evolutionary biology is a tree showing the inferred evolutionary relationships among various biological species based upon similarities and differences in their genetic traits. Evolutionary systematics uses the similarities and differences that can be observed among the species in a group under study to estimate their ancestor relation. Such a basis for comparison is called a character, and the groups that result are called its character states. The character states are arranged into a character state tree to indicate where speciation events associated with the observed changes are hypothesized to have occurred. These hypotheses are expressed as an ancestor relation diagram in which the character states play the role of individual species, and a change from one state to another represents a speciation event on the phylogenetic tree. By mid 20th century, some natural scientists realized that some pairs of such hypotheses based on different bases for comparison could be logically incompatible, i.e., they could not both be true, and they began to develop tests for, and ways to resolve, incompatibilities to estimate the ancestor relation from these hypotheses. In Chapter 1, George F. Estabrook reviews the basic concepts of character compatibility analysis, gives a survey of related work, especially McMorris' contributions to character compatibility analyses, reviews some of their applications, and presents ideas for future applications.

In biometrics, psychology, ecology, and various other fields of science, data is often summarized in a 2×2 table. In general, such tables arise when two objects/ outcomes are compared on the basis of the presence/ absence of a set of attributes. For example, in ecology two species could be compared on the basis of genetic encodings, in psychology and biometrics the table could store the dichotomous response of two observers, in epidemiology it could be the results of a clinical trial of two variants of a vaccine, and so on. In many applications, there is a need to summarize such data by a single relational statistic. In Chapter 2, W.J. Heiser and M.J. Warrens give a broad overview of such statistics, and how the different statistics may be interpreted in the contexts of various other statistics. Their discussion should be of interest to practitioners in deciding what statistic to use, and to theoreticians for studying properties of these statistics.

Graphs are ubiquitous in modern applied mathematics since they can model any binary (pairwise) relationship. For example, protein interaction graphs, where the two proteins are related if they interact in some specific biochemical process. In Chapter 3, T. McKee discusses intersection graphs, a paradigm for deriving graphs as intersection of a family of mathematical objects: vertices correspond to the objects and edges corresponds to pairwise intersecting objects. In particular, McKee surveys results related to spanning subgraphs of such intersection graphs and their new applications in computational biology and in combinatorial probability. Certain kinds of spanning trees of the protein interaction graph (represented as a particular intersection graph) are applied to show how proteins enter and leave cellular processes. At the other end of the spectrum of applications, construction of spanning subgraphs for intersection graph representations of a family of sets are applied to generalize the Inclusion-Exclusion formula or Bonferroni-type probabilistic inequalities when cardinalities of only certain kinds of intersections of the sets are known.

A common problem in any service-related organization is the decision on where to locate their facilities, such as shipping centers, shopping malls, fire stations, elementary schools, etc., so that they can optimally serve those who benefit from them while minimizing costs. The location of such facilities is discussed in context of certain networks - transportation, communication, etc. - that connect populations centers, manufacturing sites, etc. In Chapter 4, F.R. McMorris, H.M. Mulder and R.V. Vohra discuss the consensus theory, as opposed to optimization, approach to solving this problem. A consensus function takes the potential locations as input and outputs are the locations that satisfy the optimality criterion. Such a "rational" function is built using a list of intuitively natural axioms. McMorris, Mulder and Vohra survey three popular location functions - center, mean, and median, and also include some new results on the median location function.

The graph structure underlying the discrete networks in the location theory above have the property that for any three locations there must be a unique location that lies on a shortest path between any pair of the three locations. Such graphs are called median graphs. These graphs include the often-used graphs in applications - trees, hypercubes, and grids. In Chapter 5, H.M. Mulder surveys the rich structural theory of median graphs and median-type structures, and their applications in Location Theory, Consensus functions, Chemistry, Biology and Psychology, Literary history, Economics and Voting Theory.

The three locations functions discussed in Chapter 4 give a consensus-theoretic generalization of a notion of "center" of a tree based on three distinct norms used for measuring distance. In Chapter 6, M.J. Pelsmajer and K.B. Reid introduce three families of central substructures of trees that generalize these notions of "centrality" - center, centroid, and median - in a tree in a graph-theoretic sense. Their new results give a theoretical framework for each of these new concepts which naturally generalize the previous classical results for center, centroid, and median of a tree as well as their other generalizations, and algorithms for finding these substructures in trees. In the closely related Chapter 8, K.B. Reid gives a thorough survey of various concepts that have been defined and studied as a measure of "central" substructure in a tree. This survey by Reid can be read as a prologue to the discussion in Chapter 6.

Consensus theory, last discussed in Chapter 4 in the context of location theory, is also a natural framework for studying voting systems. In 1952, Kenneth May showed that the simple majority rule is the only two-candidate election procedure in which each candidate is treated equally, each voter is treated equally, and a candidate is never hurt by receiving more votes - three very natural axioms or rules for consensus. In Chapter 7, R.C. Powers discusses various generalizations of the majority rule, as based on various natural axioms that should be satisfied by the consensus function representing the voting procedure, that have been studied in the sixty years since May's seminal result.

In Chapter 9, F.S. Roberts describes and discusses many variants of an optimization model for the problem of scheduling the unloading of waiting ships at a port that takes into account the desired arrival times, quantities, and designated priorities of goods on those ships, when the said port needs to be reopened after closure due to a natural disaster or terrorist event or domestic dispute. Roberts discusses the subtleties involved in defining the objective function, and the algorithmic challenges involved in the solution, and he surveys the related literature.

In Chapter 10, D.G. Saari takes us back to consensus theory as applied to voting systems. Starting with the famous Arrow's theorem that dictatorship is the only voting rule that satisfies certain natural axioms, many important results in consensus theory state that it is impossible to have reasonable consensus functions that satisfy certain natural and innocuous properties (axioms). Saari gives an accessible discussion of what lies at the root of these obstacles to consensus, suggests a way around such difficulties by defining appropriate compatibility conditions, and illustrates his conclusions using simple examples from voting theory.

We conclude with Chapter 11, in which G. Estabrook, T.A. McKee, H.M. Mulder, R.C. Powers and F.S. Roberts give a survey of Buck McMorris' work over the past forty years in discrete mathematics, especially Evolutionary Biology, Intersection Graph Theory, Competition Graphs and related phylogeny graphs, Location Functions on Graphs, and Consensus Theory. This survey helps unify the themes explored in the previous chapters of this book under the aegis of wide-ranging scholarship of Buck McMorris in Discrete Applied Mathematics.