MATH 380

Hemanshu Kaul

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A competitive Species Model with carrying capacity

Blue whales and Fin whales are competing species in the same seas.

Let $x_1 = B =$ # Blue whales , $x_2 = F =$ # Fin whales.

$g_B =$ overall growth rate of blue whales (per year)

$g_F =$ overall ——— " ——— Fin —"— (per year)

$c_B =$ competition factor affecting blue w. (whales per year)

$c_F =$ ——————— " ——————— fin w. (whales per year)

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In the sea we are studying, upto 150000 Blue whales can be be supported by the environment, and upto 400000 fin whales. And, its been observed B have 5% p.a. intrinsic growth rate

& F have 8% ———— " ———— .

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We model g_B & g_F as a (scaled) logistic model:

$$g_B = 0.05\, x_1 \left(1 - x_1/150000\right)$$
$$g_F = 0.08\, x_2 \left(1 - x_2/400000\right)$$

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We model the competition between the species as being proportional to the number of interactions between them.

$$C_B = \alpha x_1 x_2 \quad , \quad C_F = \alpha x_1 x_2 \ ,$$ where $\alpha > 0$ is a constant.

use of same α indicates what underlying assumption?

A competitive Species Model with carrying capacity

Blue Whales and Fin Whales are competing species in the same seas.

Let $x_1 = B = \#$ Blue whales , $x_2 = F = \#$ Fin Whales .

$$\frac{dx_1}{dt} = (0.05)\, x_1 \left(1 - x_1/150000\right) - \alpha x_1 x_2$$

$$\frac{dx_2}{dt} = (0.08)\, x_2 \left(1 - x_2/400000\right) - \alpha x_1 x_2 \quad , \quad \alpha > 0$$

constant (unknown)

Due to hunting & environmental effects, current populations are $x_1(0) = 5000$ B. whales & $x_2(0) = 70000$ F. whales.

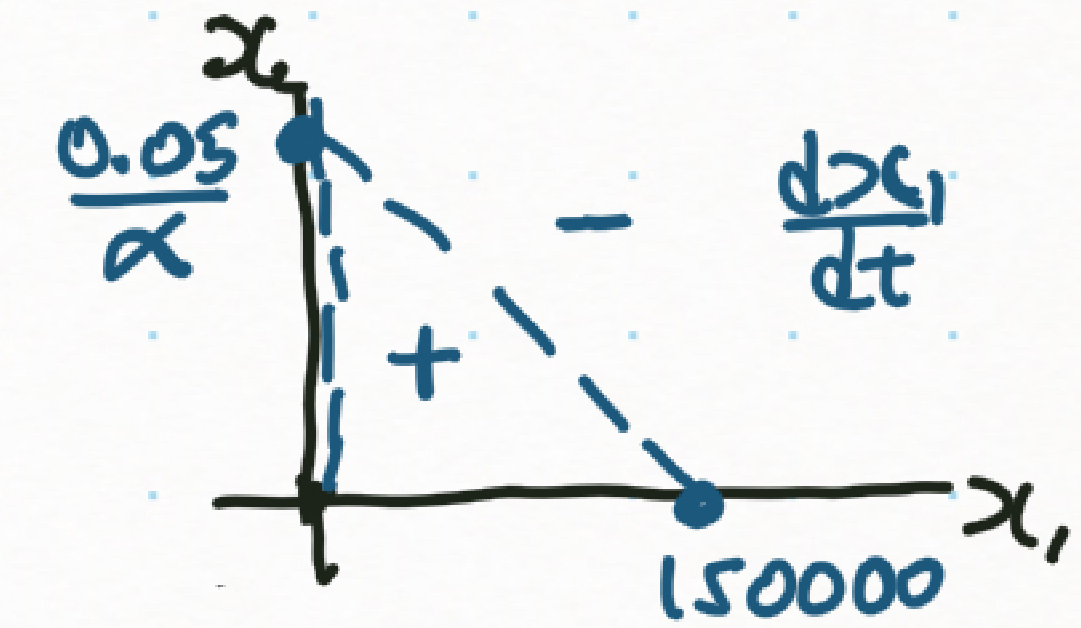
How will the two populations change over the short-term? long-term?

Will Blue Whales become extinct? etc.

Phase Plane

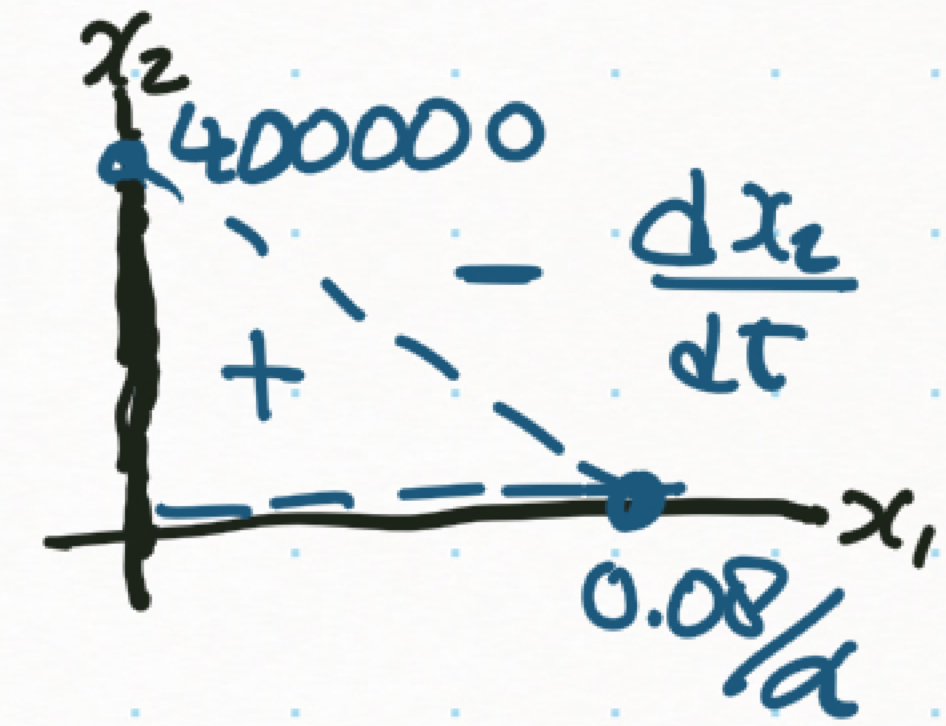
$$\frac{dx_1}{dt} = 0 \iff 0 = x_1\left(0.05 - \frac{0.05}{150000}x_1 - \alpha x_2\right)$$

i.e., $x_1 = 0$ or $\dfrac{x_1}{150000} + \dfrac{\alpha}{0.05}x_2 = 1$ ← eqn. of a line



$$\frac{dx_2}{dt} = 0 \iff 0 = x_2\left(0.08 - \frac{0.08}{400000}x_2 - \alpha x_1\right)$$

i.e., $x_2 = 0$ or $\dfrac{x_2}{400000} + \dfrac{\alpha}{0.08}x_1 = 1$



Combine these together in a single phase plane.

Phase Plane

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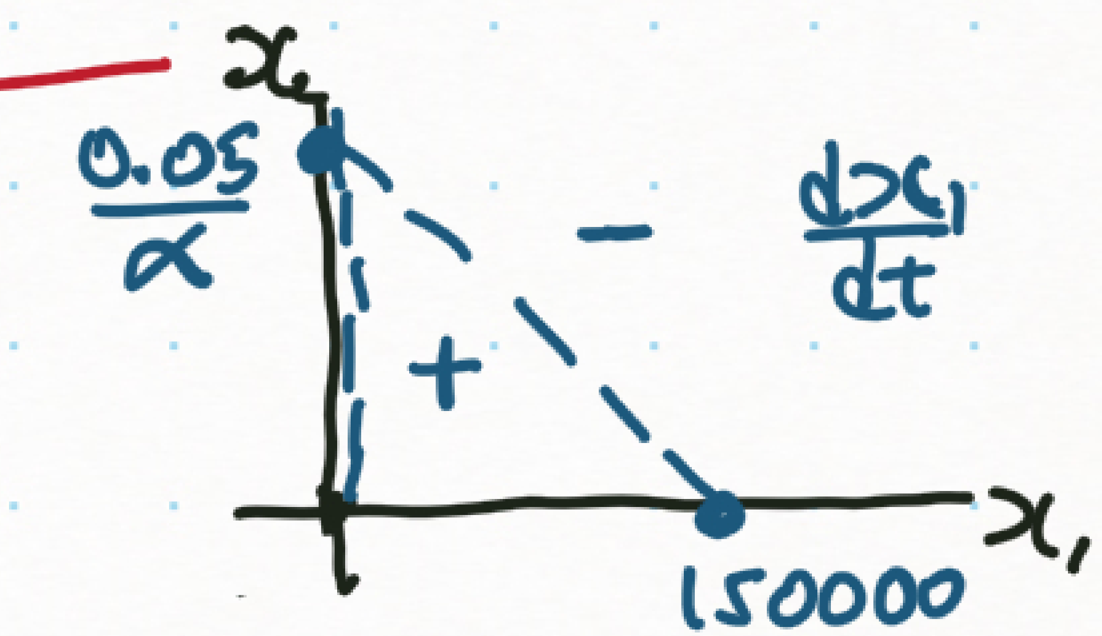
Line 1



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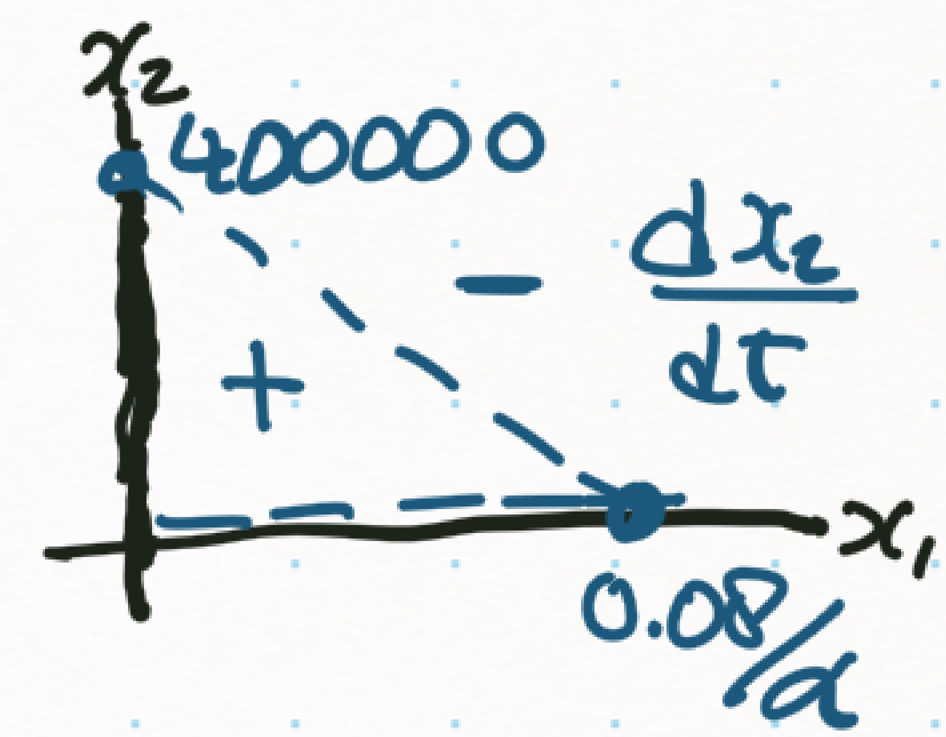
Line 2



4 possibilities based on $\frac{0.05}{\alpha}$ < or > 400000

& $\frac{0.08}{\alpha}$ < or > 150000

(Combine these together in a single phase plane.

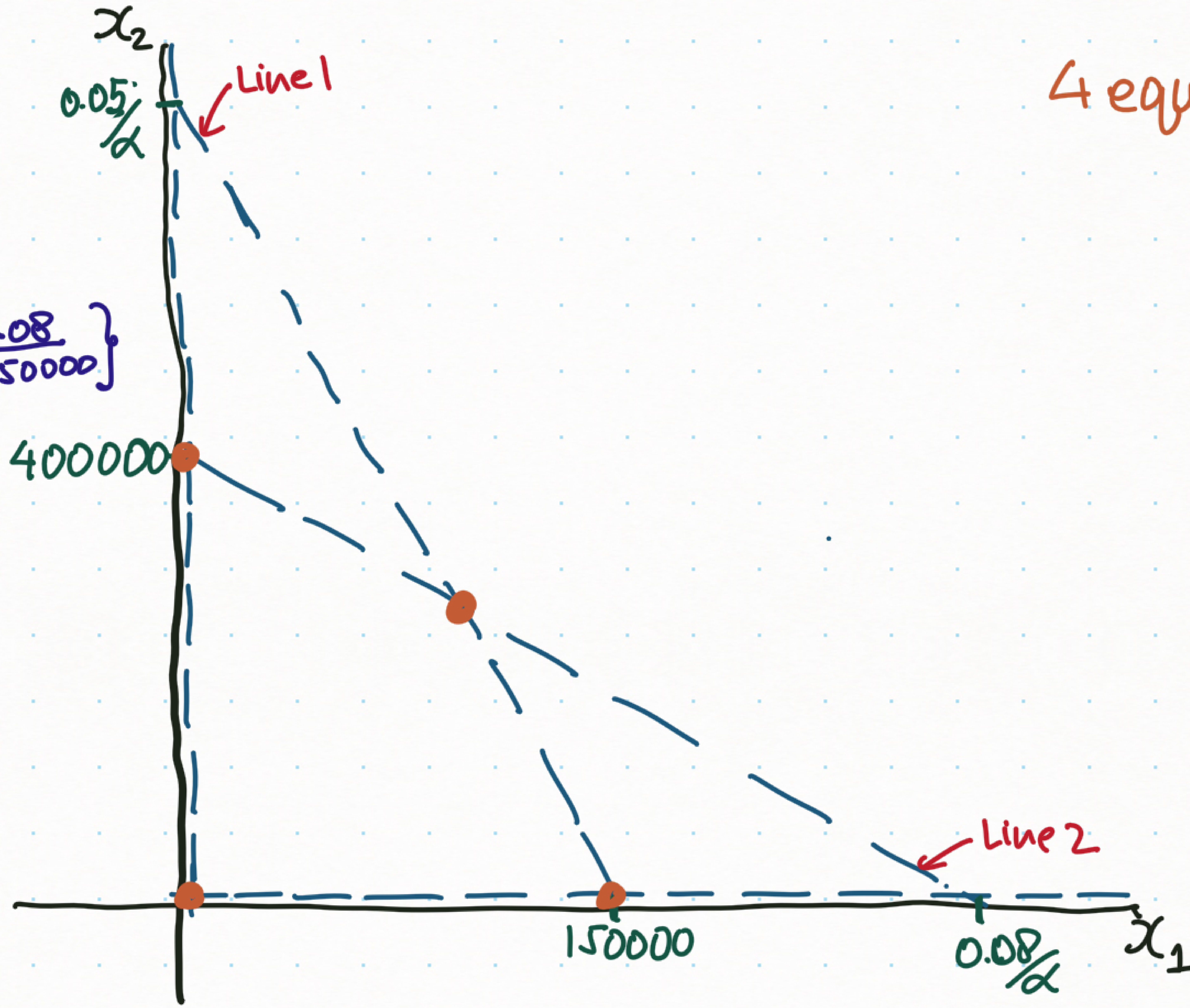
Assuming

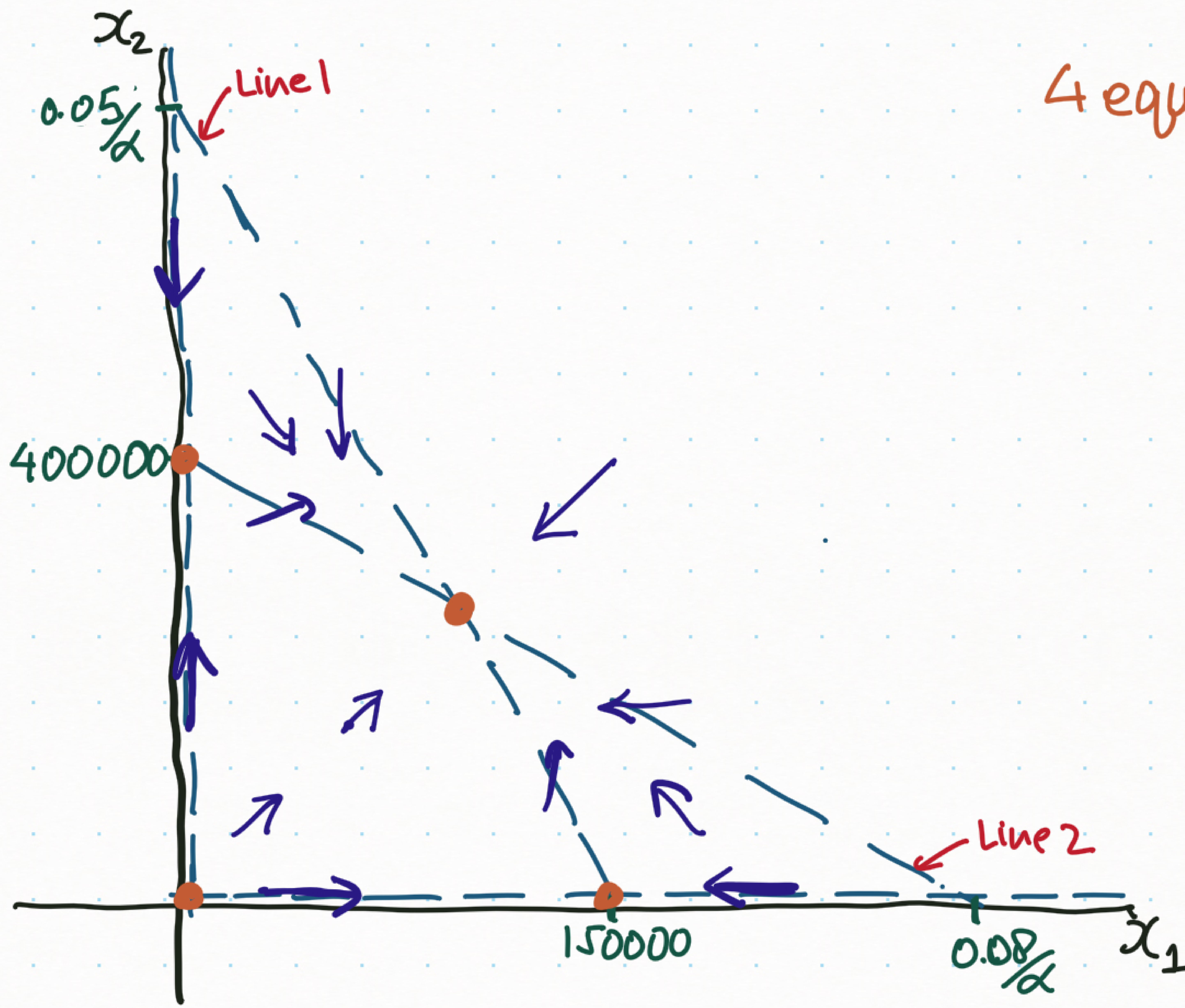
$\frac{0.05}{\alpha} > 400000$ &

$\frac{0.08}{\alpha} > 150000$

i.e, $\alpha < \min\left\{\frac{0.05}{400000}, \frac{0.08}{150000}\right\}$

4 equillibrium points





4 equillibrium points

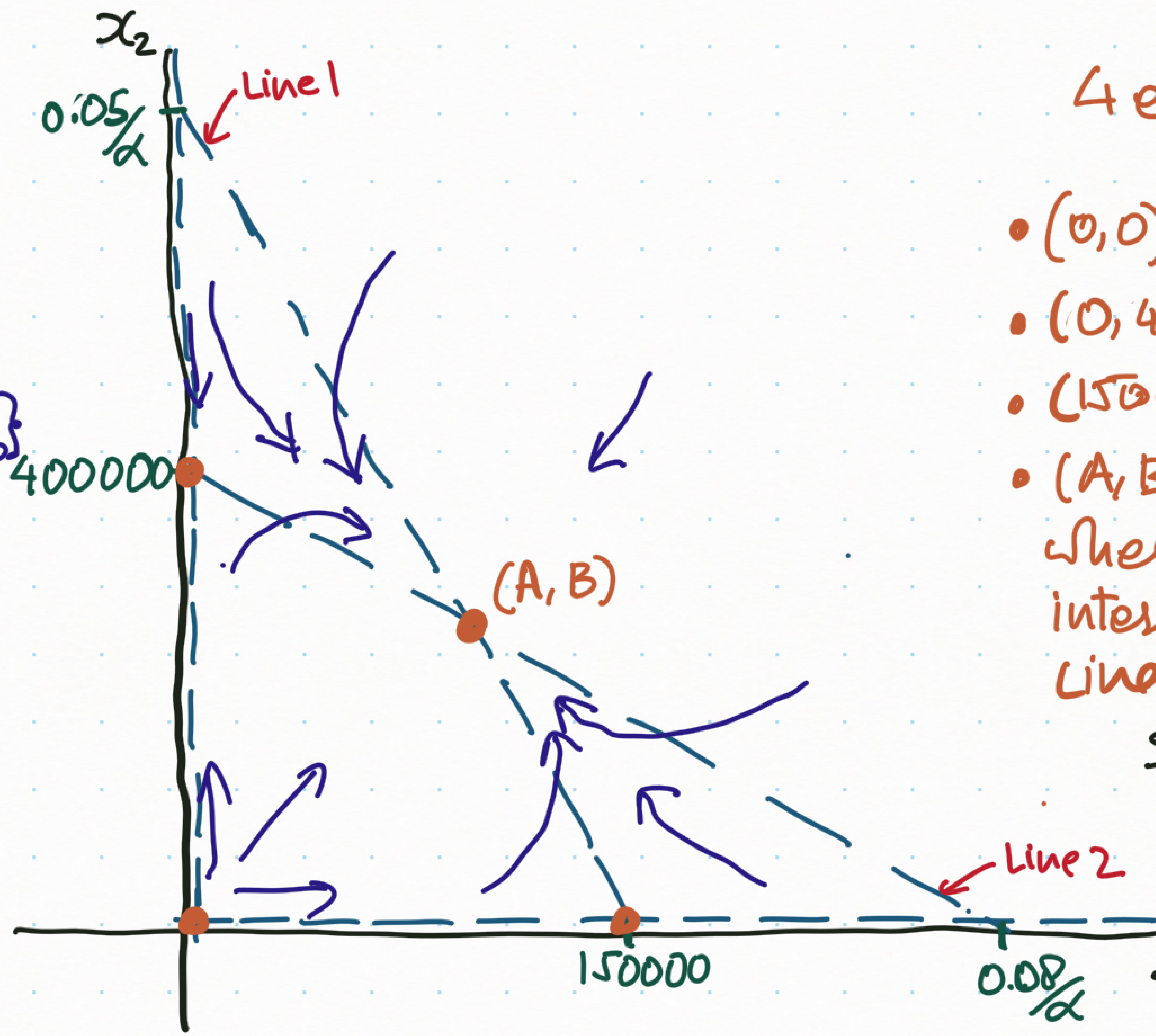
Assuming

$$\frac{0.05}{\alpha} > 400000$$

$$\frac{0.08}{\alpha} > 150000$$

ie

$$\alpha < \min\left\{\frac{0.05}{400000}, \frac{0.08}{150000}\right\}$$



4 equillibrium points

- (0,0) unstable
- (0, 400000) unstable
- (150000, 0) unstable
- (A, B) stable.

where (A, B) is the intersection of Line 1 & Line 2

Solve

$$\frac{x_1}{150000} + \frac{\alpha}{0.05} x_2 = 1$$

$$\frac{x_2}{400000} + \frac{\alpha}{0.08} x_1 = 1$$

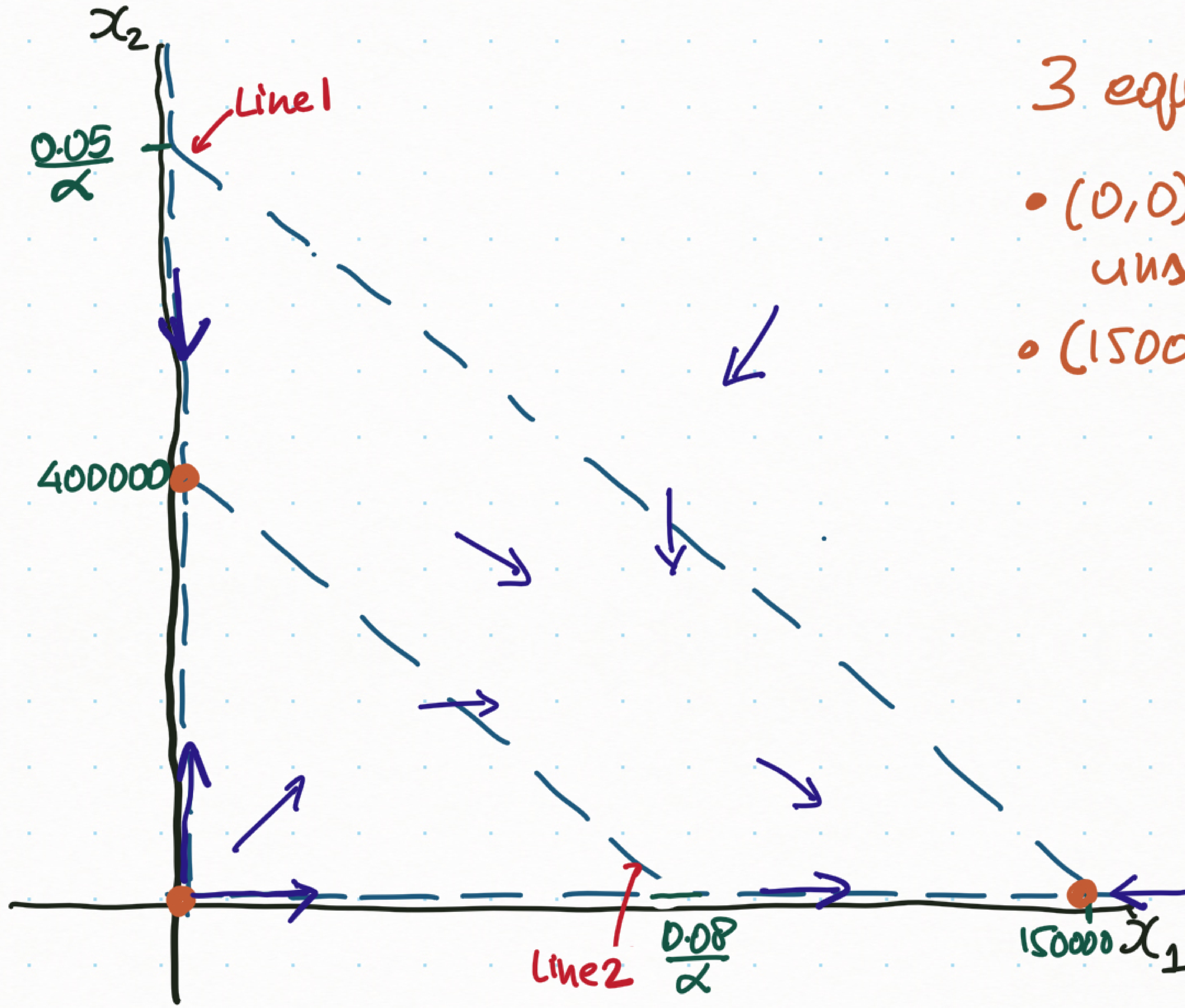
if α is known.

Long-term: Both populations move towards peaceful coexistence, starting from $x_1(0) = 5000$, $x_2(0) = 70000$

Assuming

$\frac{0.05}{\alpha} > 400000$ &

$\frac{0.08}{\alpha} < 150000$



3 equilibrium pt.s

- (0,0) & (0,400000) unstable
- (150000,0) stable

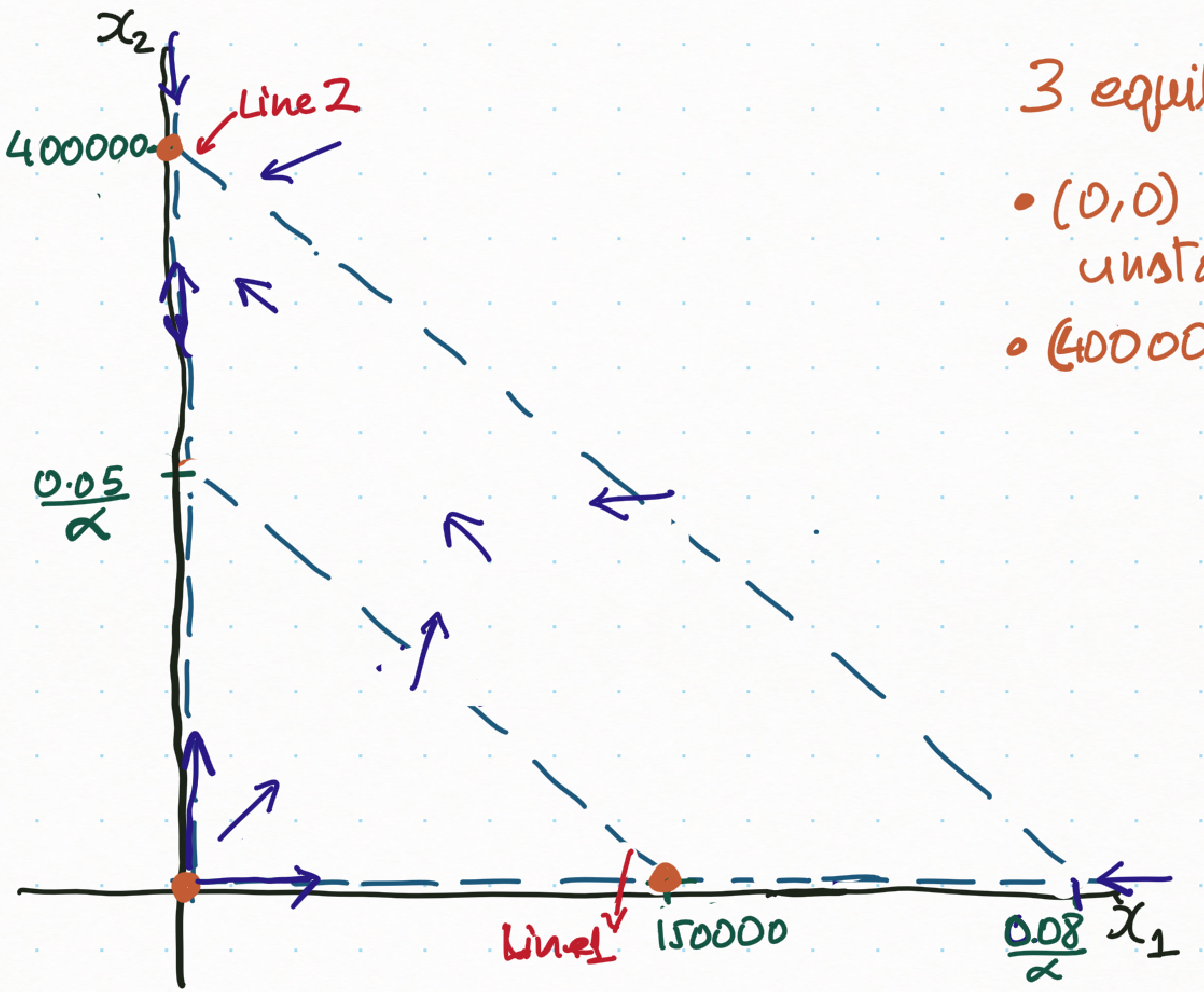
Long-term: Fin Whales die out while Blue Whales stabilize at 150000

Unlikely to happen

Assuming

$\frac{0.05}{\alpha} < 400000$ &

$\frac{0.08}{\alpha} > 150000$



3 equilibrium pt.s

- (0,0) & (0,150000) unstable
- (400000,0) stable

Long-term: Blue Whales die out while Fin Whales stabilize at 400000

Unlikely to happen

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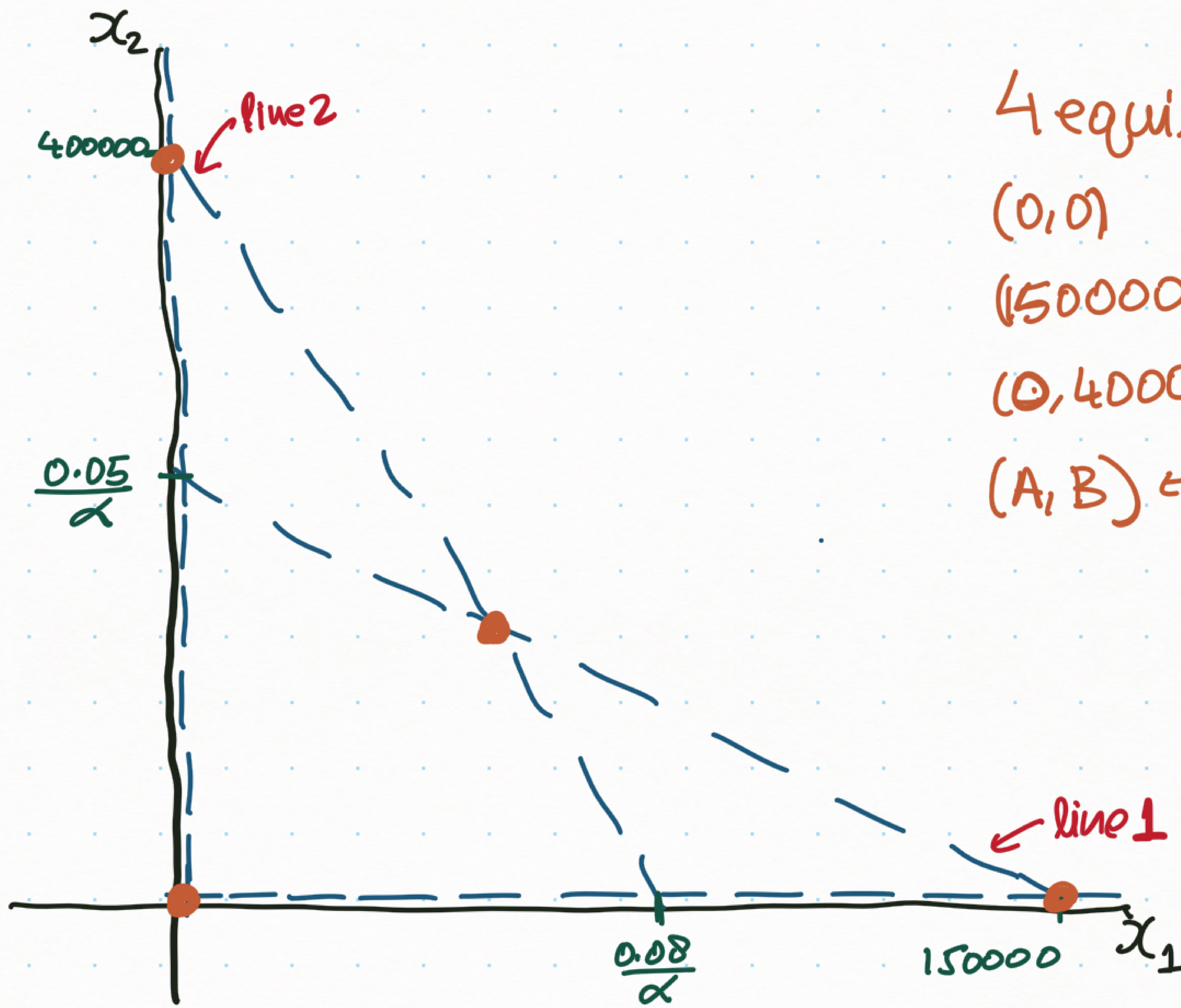

4 equilibrium pt.s

(0, 0)

(150000, 0)

(0, 400000)

(A, B) ← intersection of lines 1 & 2.

Assuming

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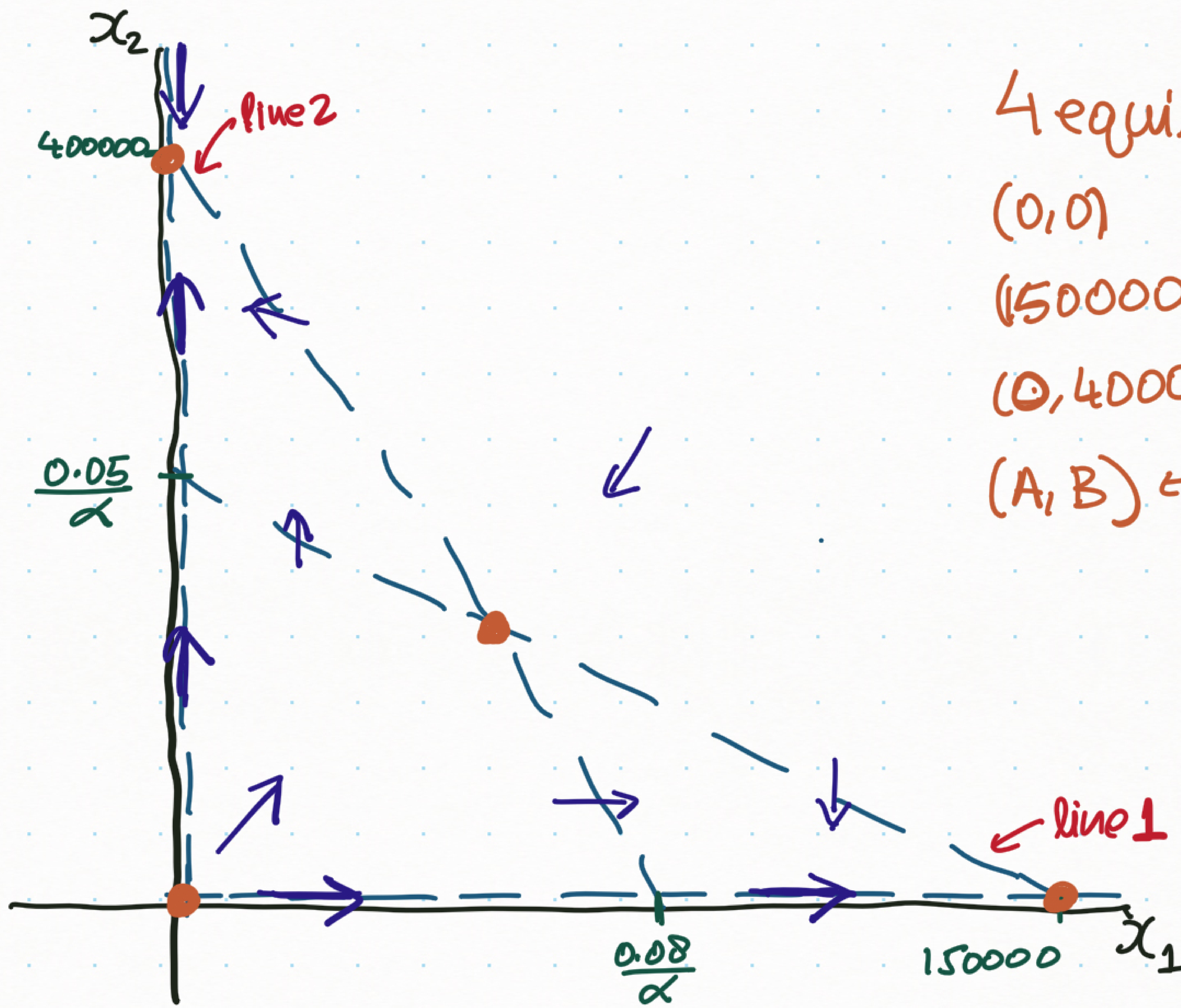

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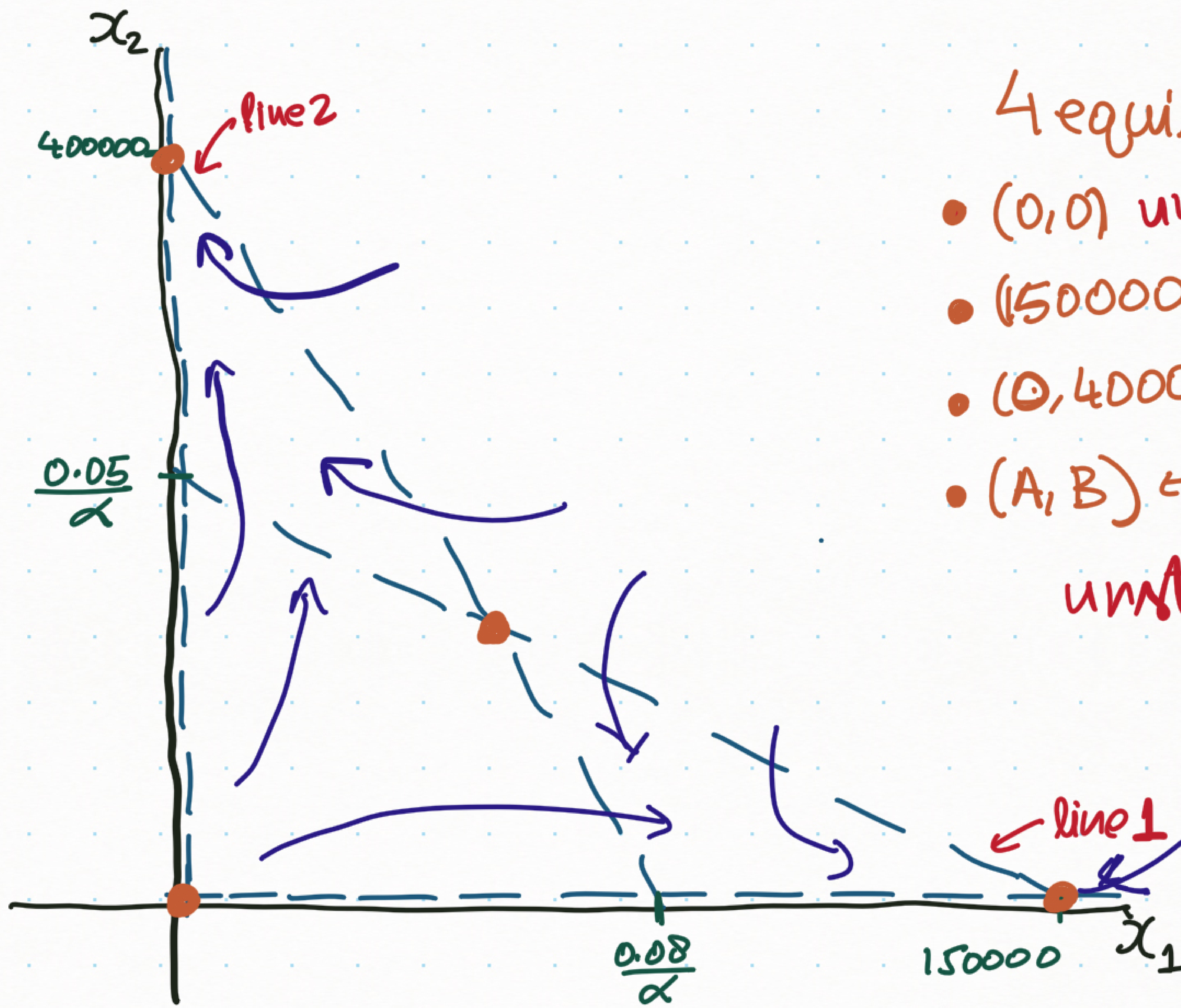


4 equilibrium pt.s

- (0,0) unstable
- (150000,0) stable
- (0,400000) stable
- (A, B) ← intersection of lines 1 & 2.

unstable

Long-Term: Either Fins or Blues die out but both do not survive.

LETS ANALYZE THIS FURTHER

Assuming

$\frac{0.05}{\alpha} > 400000$

$\frac{0.08}{\alpha} > 150000$

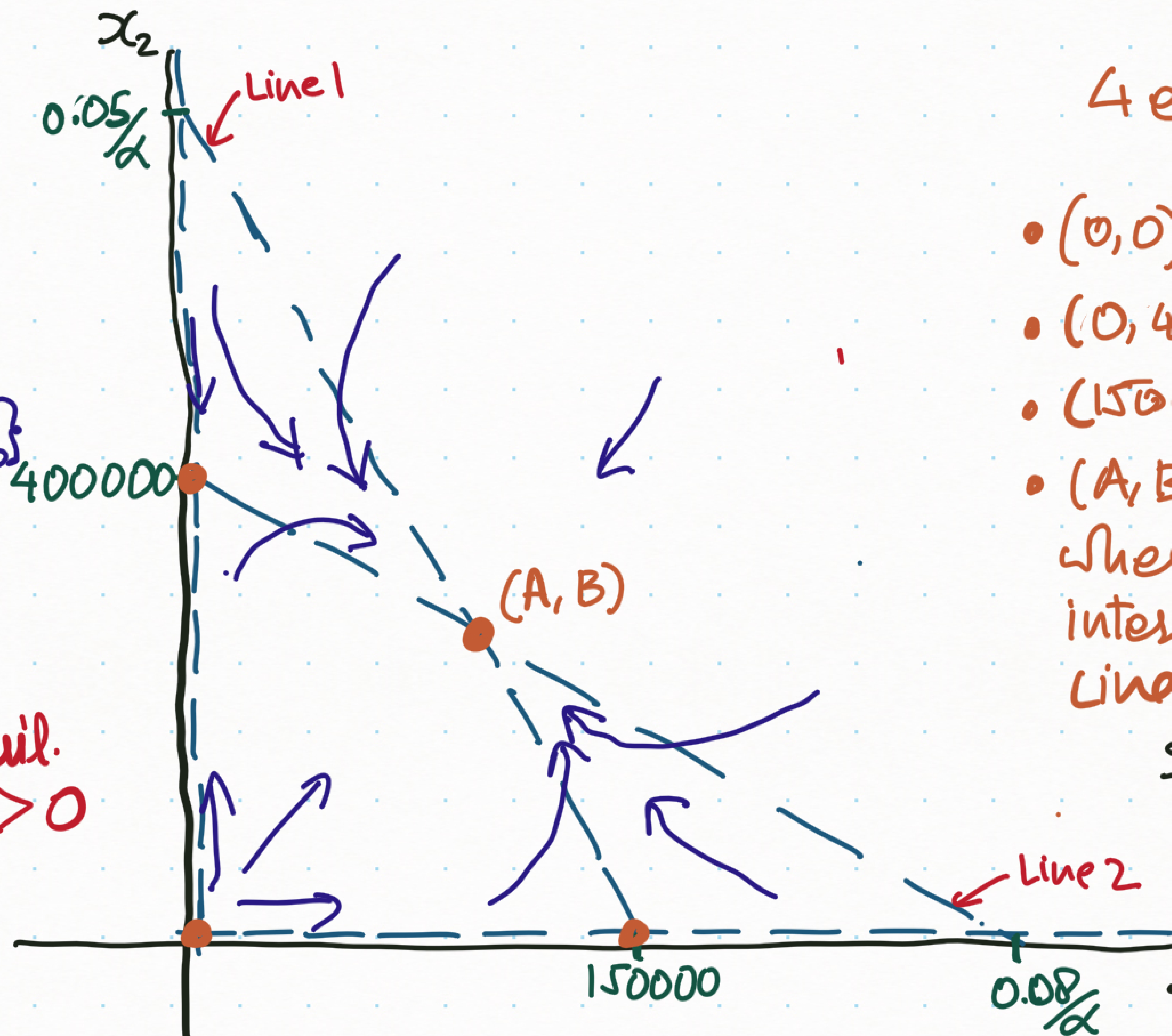
ie

$\alpha < \min\left\{\frac{0.05}{400000}, \frac{0.08}{150000}\right\}$

i.e.,

$\alpha < 1.25 \times 10^{-7}$

gives a stable equil. with $x_1 > 0$ & $x_2 > 0$



4 equillibrium points

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- (A, B) stable.

where (A, B) is the intersection of Line 1 & Line 2

Solve

$\frac{x_1}{150000} + \frac{\alpha}{0.05} x_2 = 1$

$\frac{x_2}{400000} + \frac{\alpha}{0.08} x_1 = 1$

if α is known.

Long-term: Both populations move towards peaceful coexistence, starting from $x_1(0) = 5000$, $x_2(0) = 70000$

In absence of any other external factors,
if $\alpha < 1.25 \times 10^{-7}$ then x_1 (Blues) & x_2 (Fins) will grow from initial populations $x_1(0) = 5000$ & $x_2(0) = 70000$ to their long-term populations.

How long will this take?

In absence of any other external factors,
if $\alpha < 1.25 \times 10^{-7}$ then x_1 (Blues) & x_2 (Fins) will grow from initial populations $x_1(0) = 5000$ & $x_2(0) = 70000$ to their long-term populations.

How long will this take? Numerical computation using

<u>Euler's Method</u> $\frac{dx}{dt} = f(x,y,t)$, $\frac{dy}{dt} = g(x,y,t)$ with initial values x_0, y_0, t_0

Fix a step size Δt (step size)

For $i = 1, 2, 3, \dots, n$

$$t_i \leftarrow t_{i-1} + \Delta t$$
$$x_i \leftarrow x_{i-1} + f(x_{i-1}, y_{i-1}, t_{i-1}) \Delta t$$
$$y_i \leftarrow y_{i-1} + g(x_{i-1}, y_{i-1}, t_{i-1}) \Delta t$$

approximate solution at
t_0 ← first time
$t_0 + \Delta t$
$t_0 + 2\Delta t$
$\vdots$
$t_0 + n\Delta t$ ← final time

we have

$$\frac{dx_1}{dt} = 0.05x_1\left(1-\frac{x_1}{150000}\right)-\alpha x_1x_2$$

$$\frac{dx_2}{dt} = 0.08x_2\left(1-\frac{x_2}{400000}\right)-\alpha x_1x_2$$

$$\Delta x_1 = \left(0.05x_1\left(1-\frac{x_1}{150000}\right)-\alpha x_1x_2\right)\Delta t$$

$$\Delta x_2 = \left(0.08x_2\left(1-\frac{x_2}{400000}\right)-\alpha x_1x_2\right)\Delta t$$

We will take $\Delta t = 1$ year

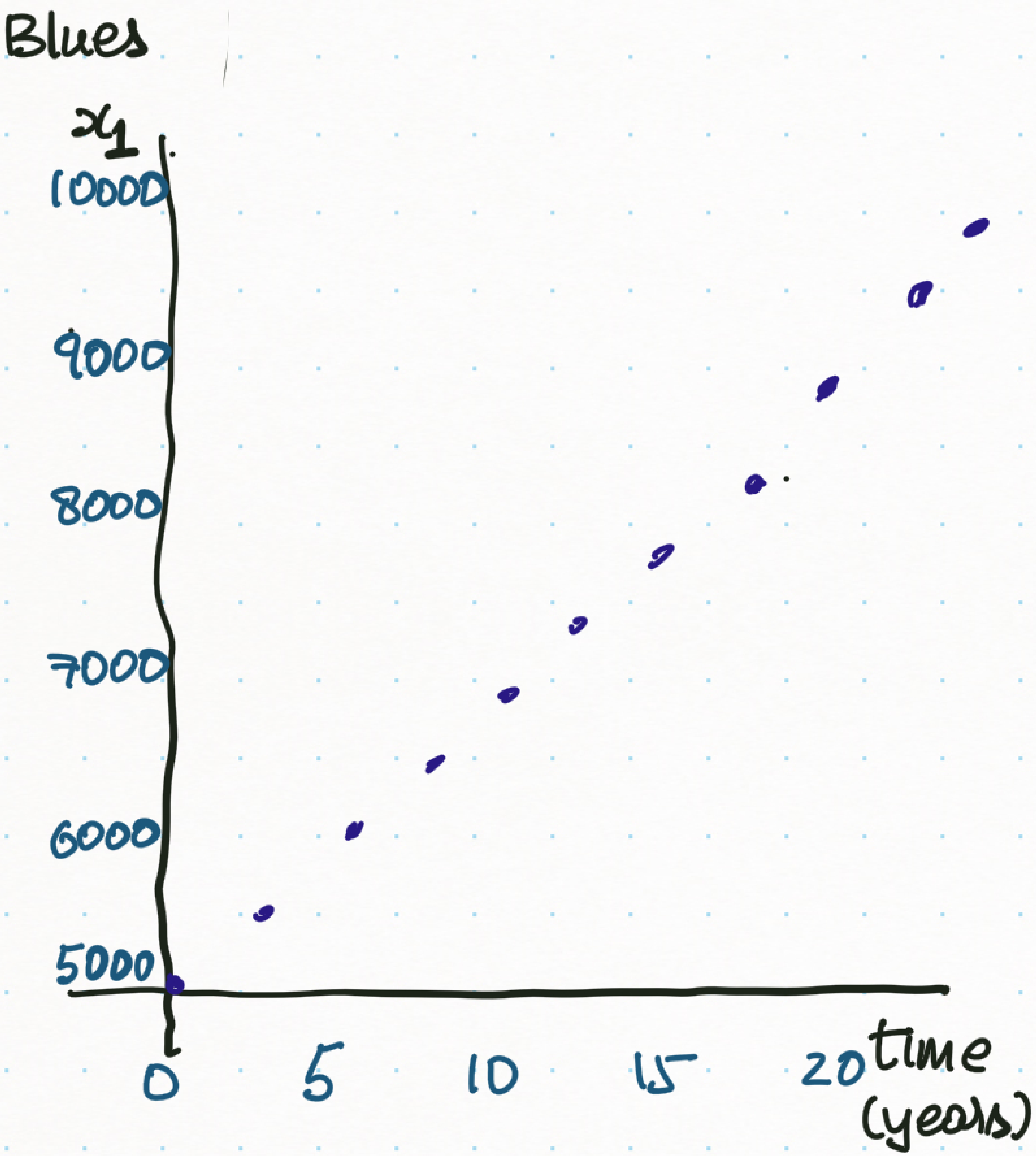
start with $x_1(0) = 5000$

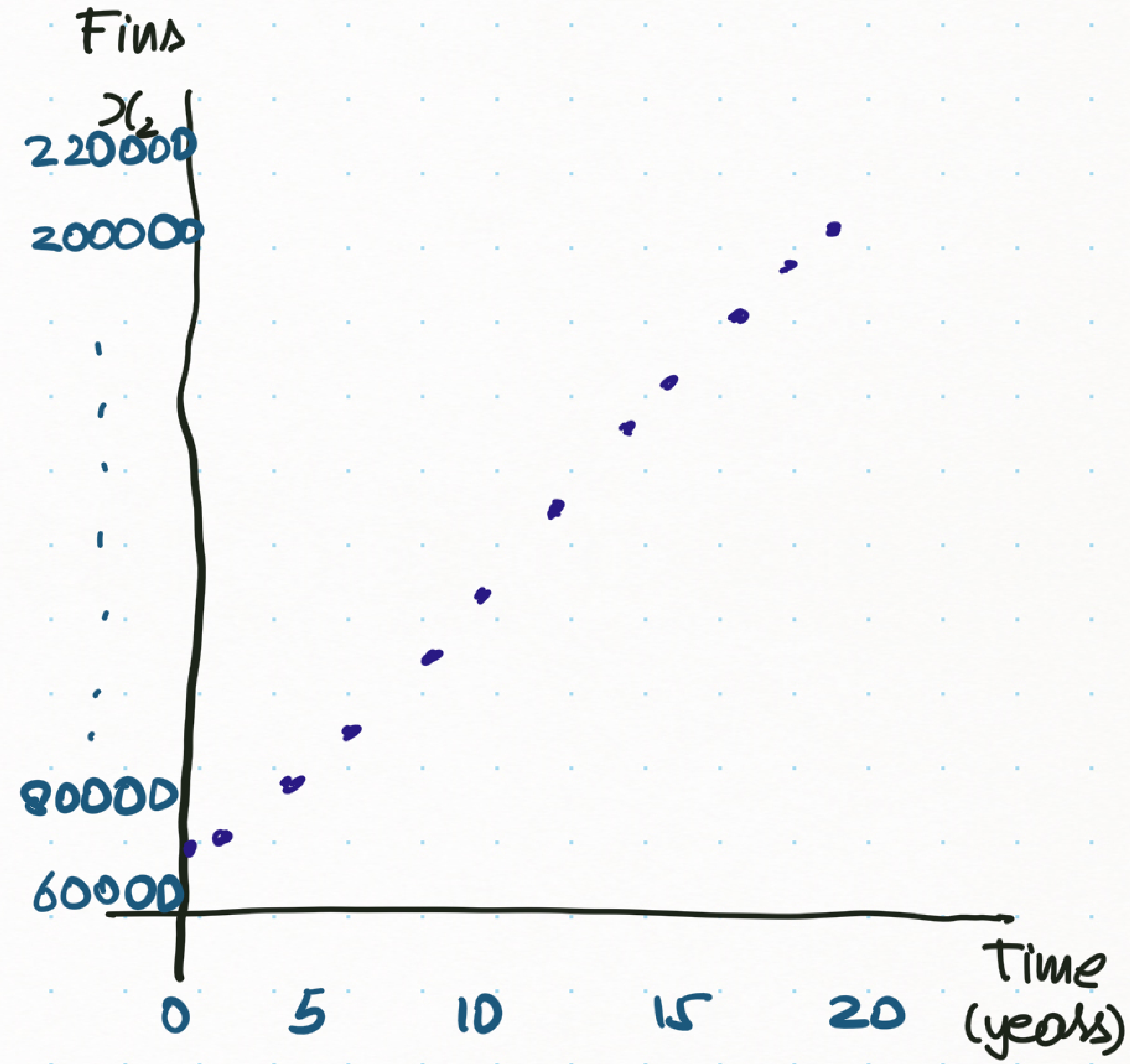
$x_2(0) = 70000$

& fix $\underline{\alpha = 10^{-7}} < 1.25\times10^{-7}$

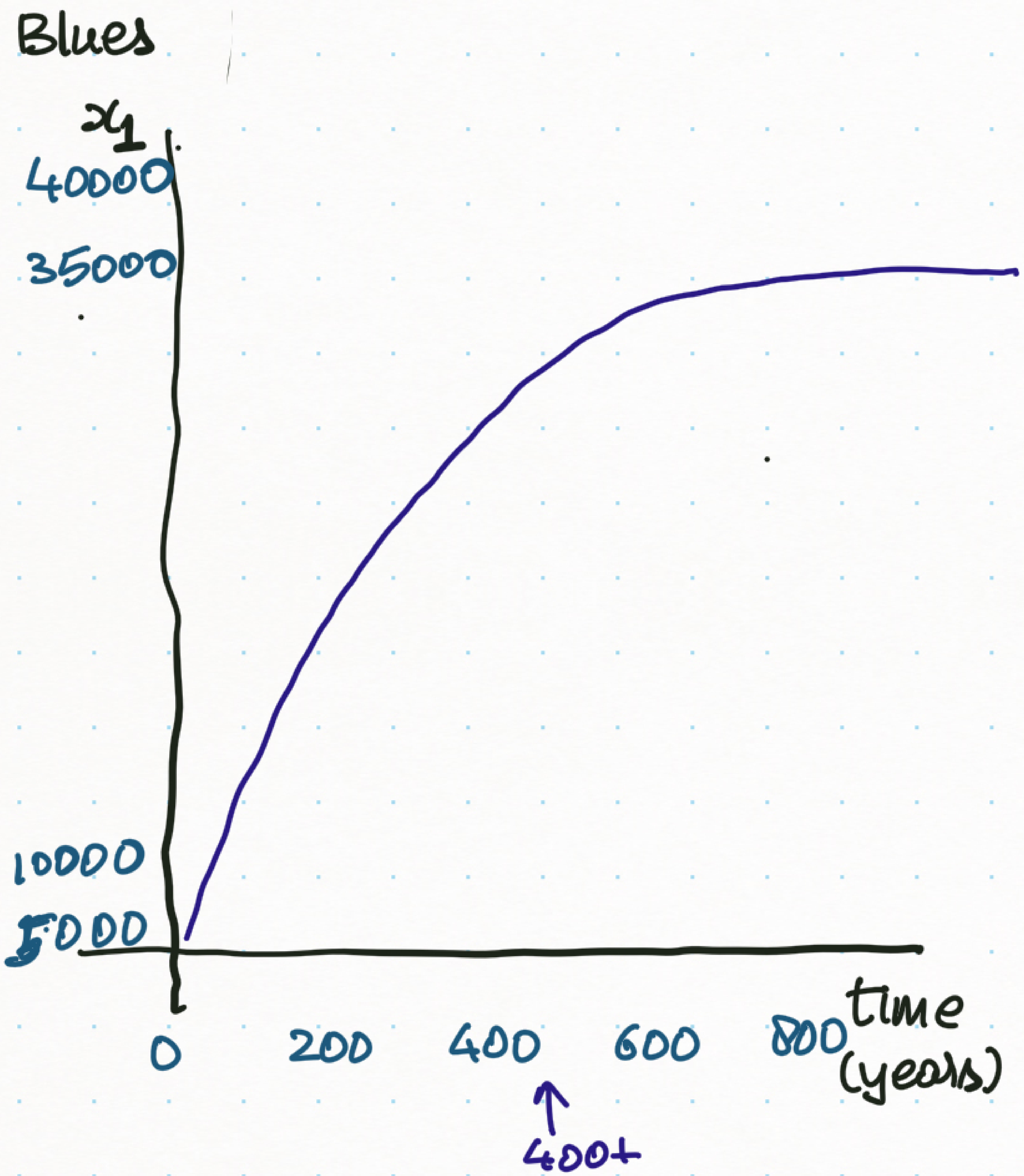
This will the stable eq. pt. as $(35294, 382352)$

$\alpha = 10^{-7}$, $n = 20$ time steps (yearly)

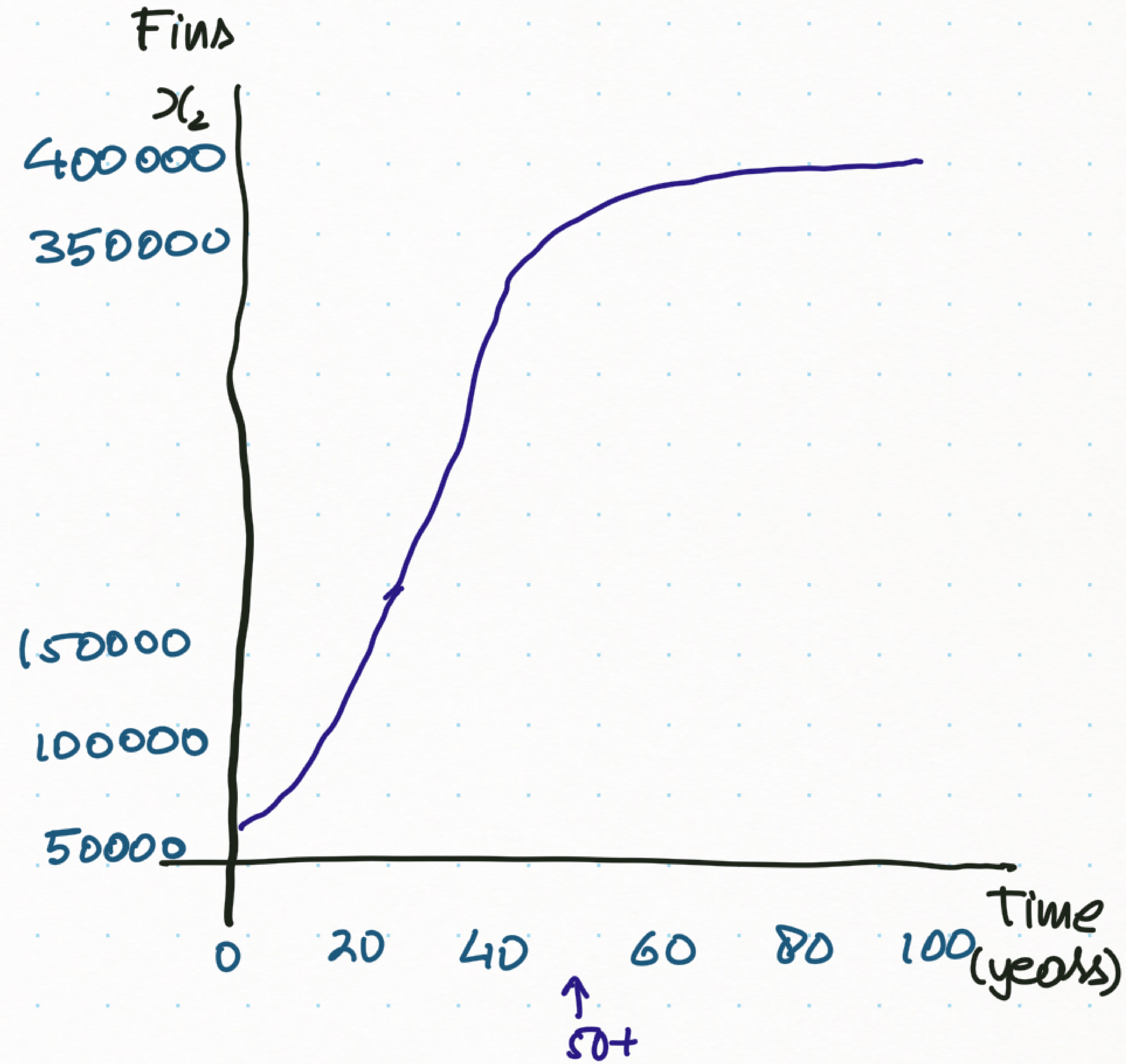




$\alpha = 10^{-7}$

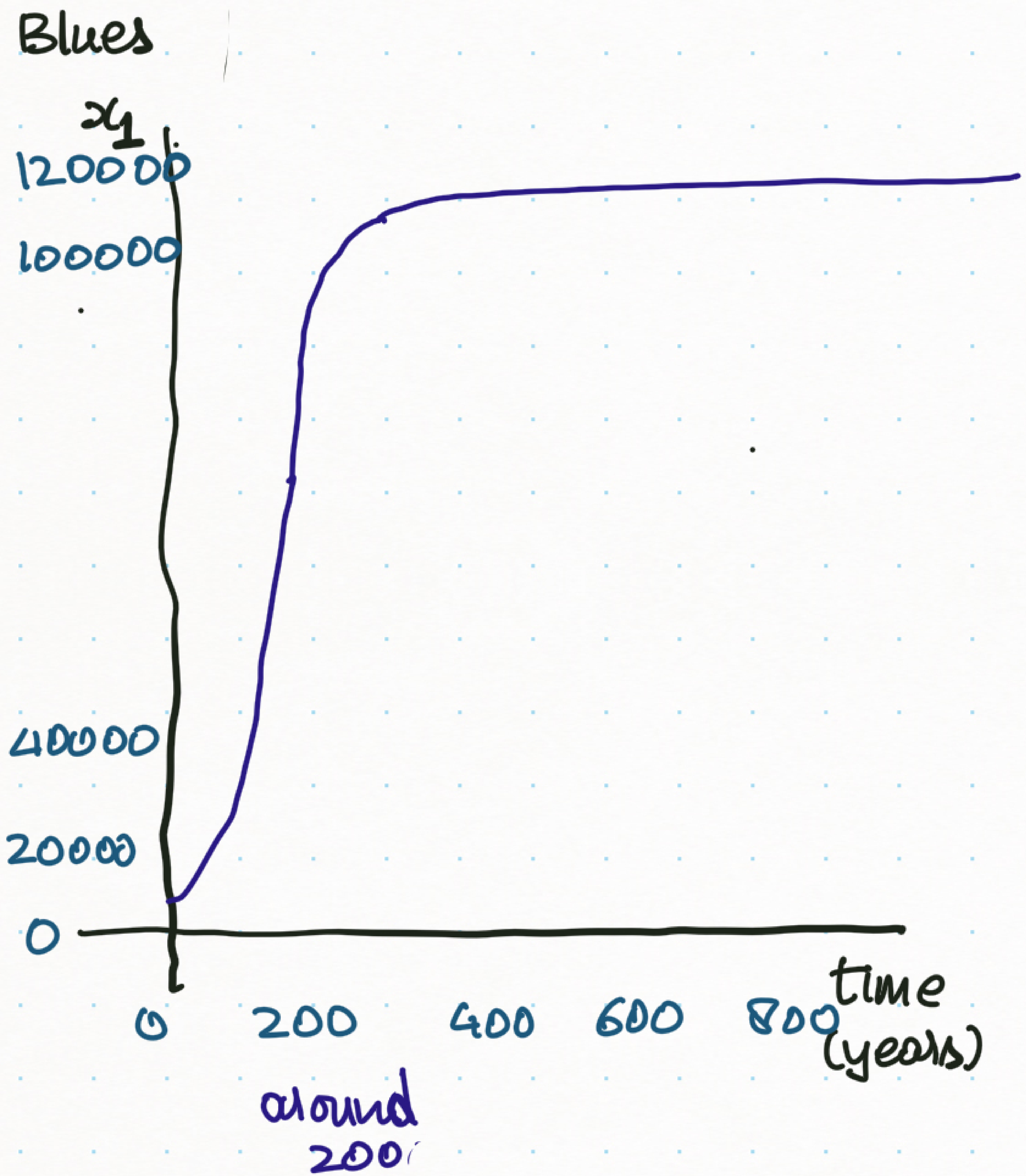


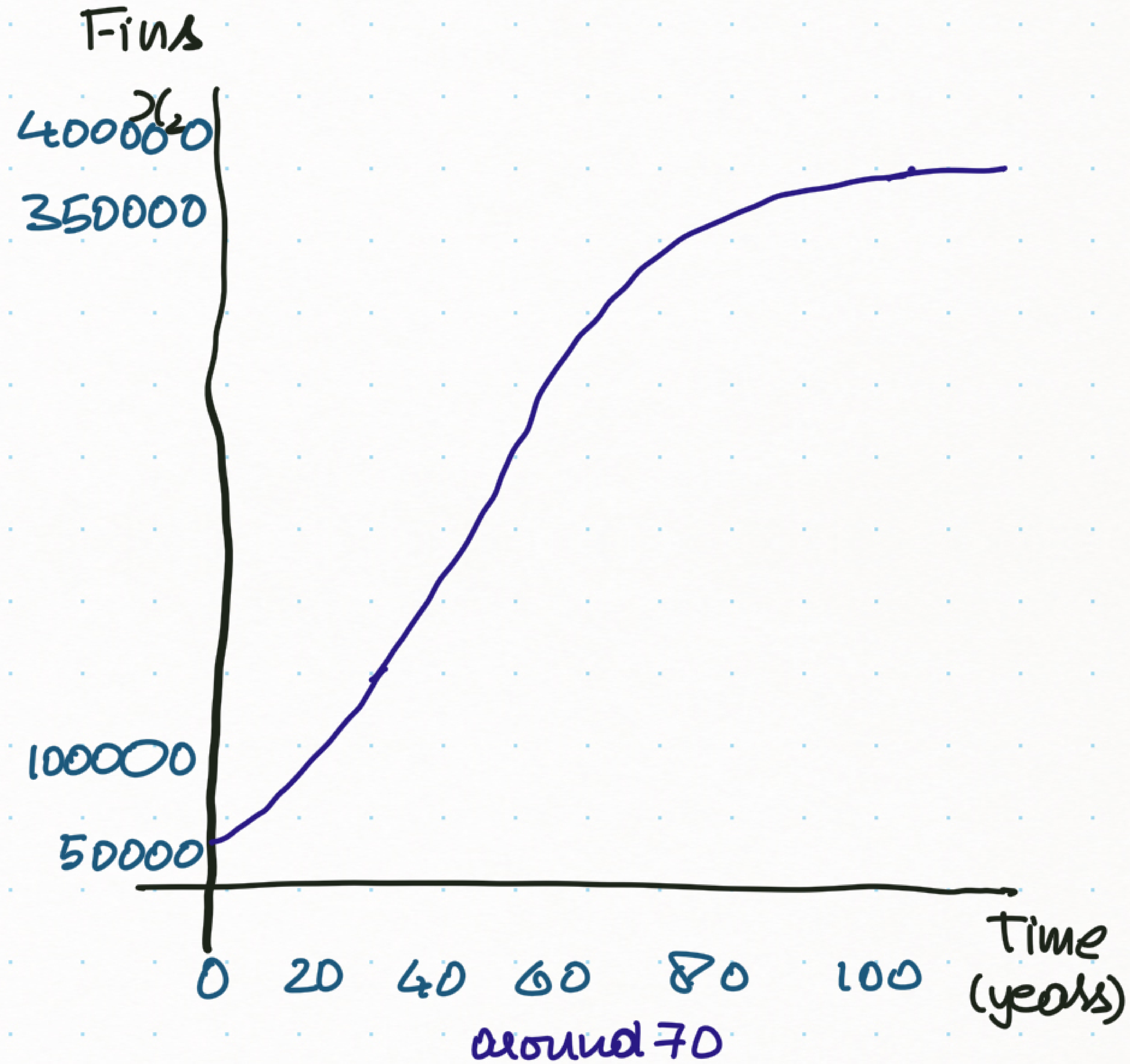
Blues
x_1
40000
35000
10000
5000
0
200
400
600
800
time (years)
400+



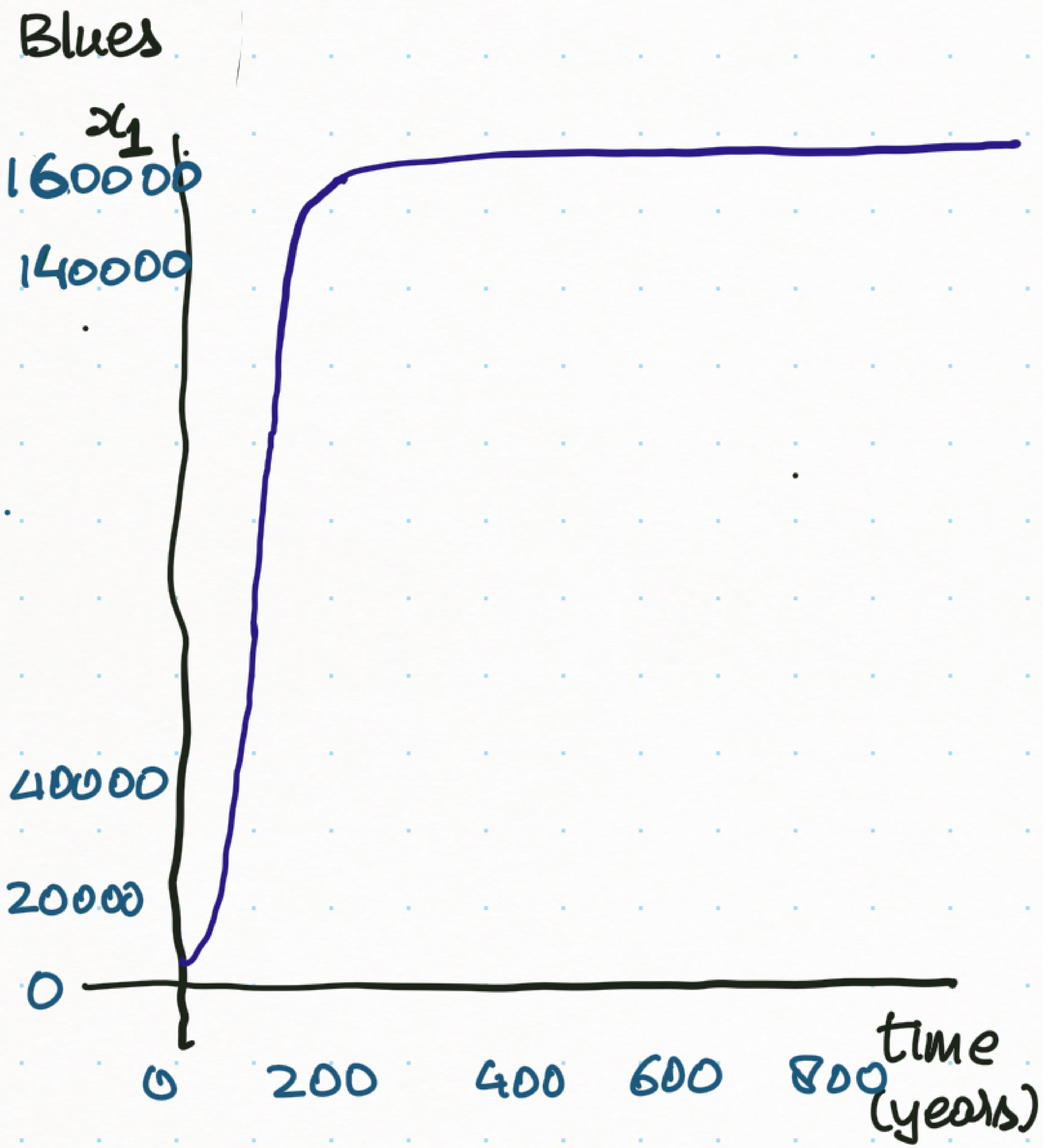
Fins
x_2
400000
350000
150000
100000
50000
0
20
40
60
80
100
Time (years)
50+

$$\alpha = 3\times10^{-8}$$

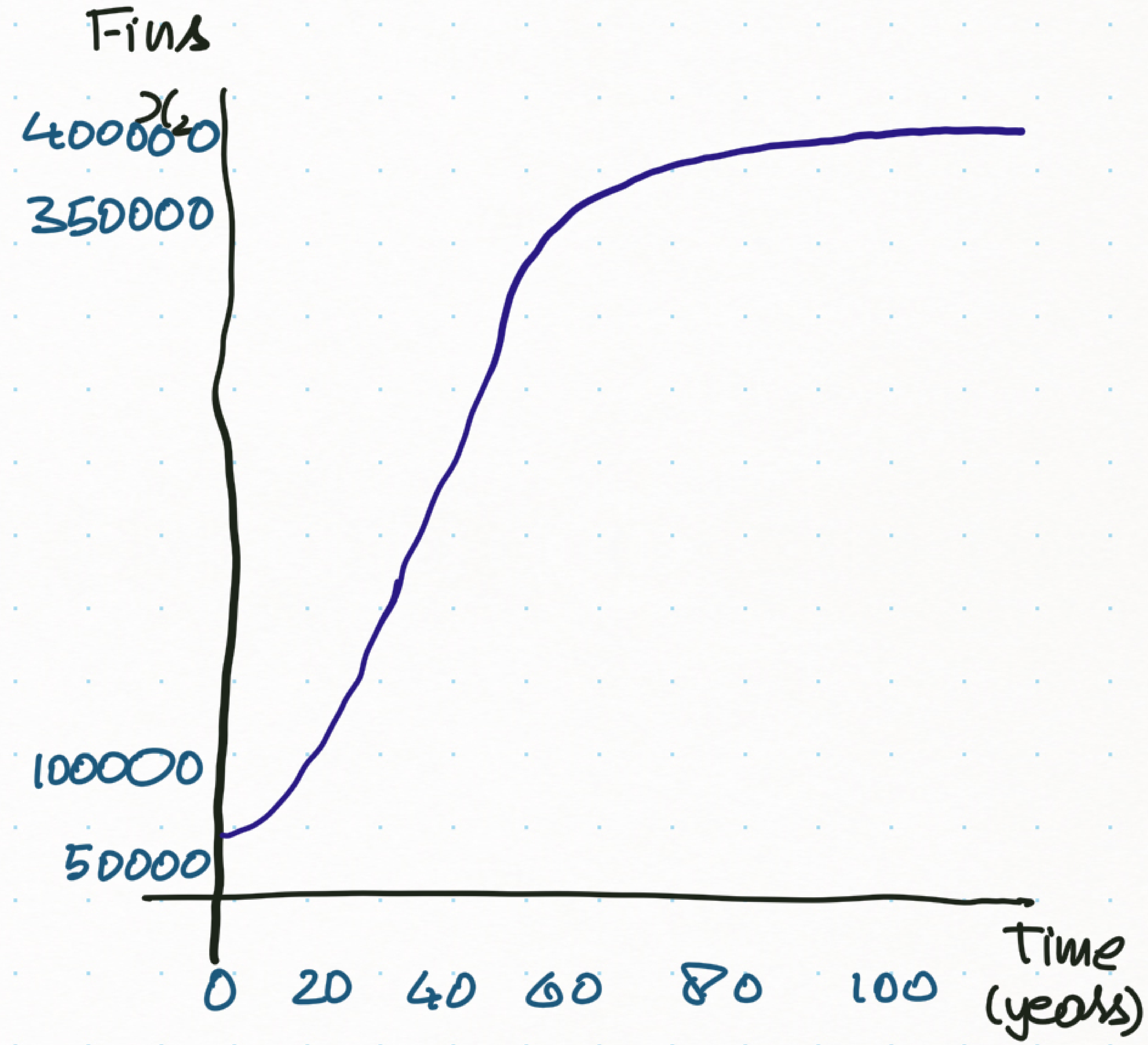




$\alpha = 10^{-9}$



around 150



around 65

we have

$$\frac{dx_1}{dt} = 0.05x_1\left(1-\frac{x_1}{150000}\right)-\alpha x_1x_2$$

$$\frac{dx_2}{dt} = 0.08x_2\left(1-\frac{x_2}{400000}\right)-\alpha x_1x_2$$

$$\Delta x_1 = \left(0.05x_1\left(1-\frac{x_1}{150000}\right)-\alpha x_1x_2\right)\Delta t$$

$$\Delta x_2 = \left(0.08x_2\left(1-\frac{x_2}{400000}\right)-\alpha x_1x_2\right)\Delta t$$

We will take $\Delta t = 1$ years

start with $x_1(0) = 5000$

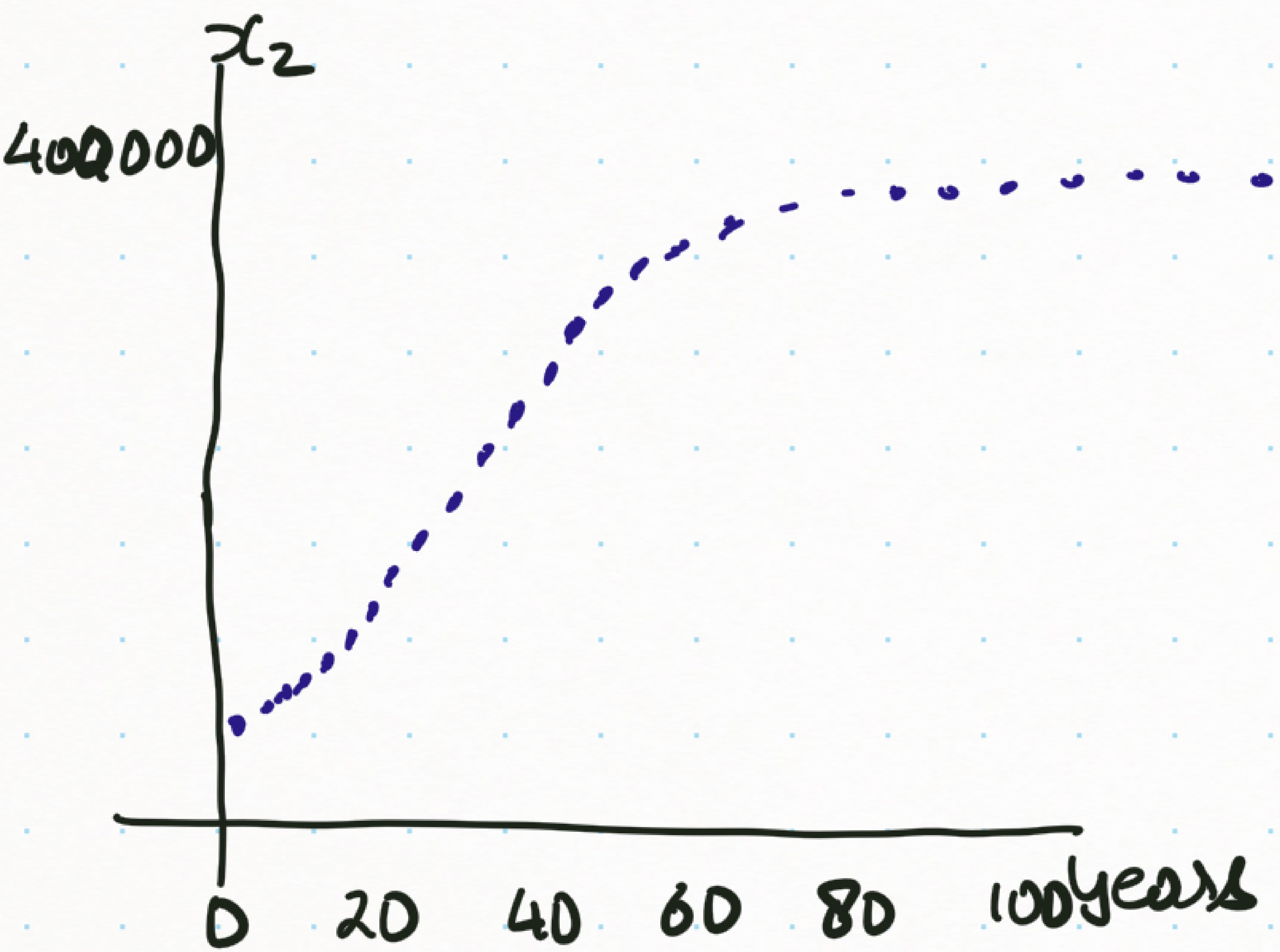
$x_2(0) = 70000$

& fix $\alpha = 10^{-7} < 1.25\times10^{-7}$

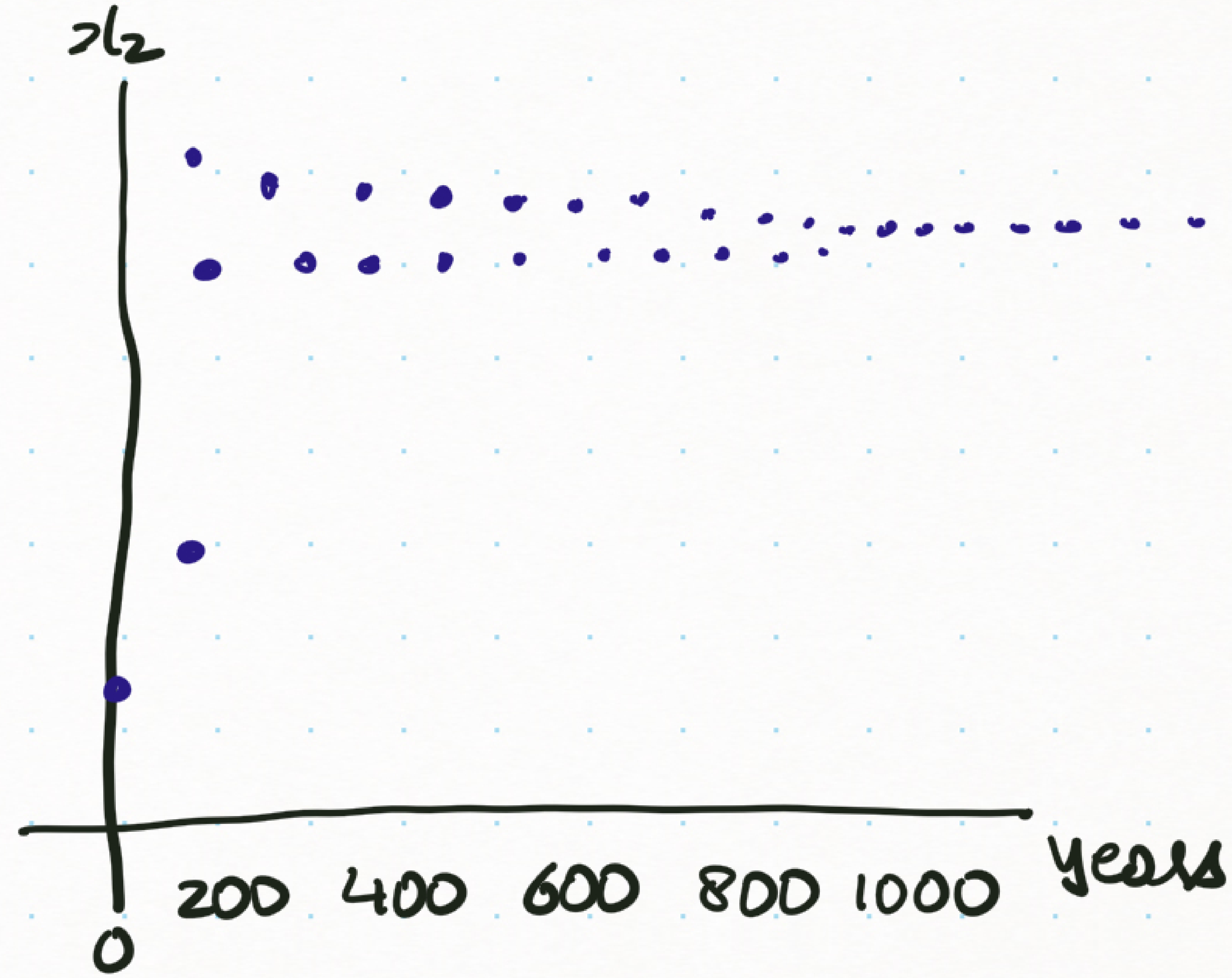
This will the stable eq. pt. as $(35294, 382352)$

What happens if we take larger step size $\Delta t = 2, \ldots$?

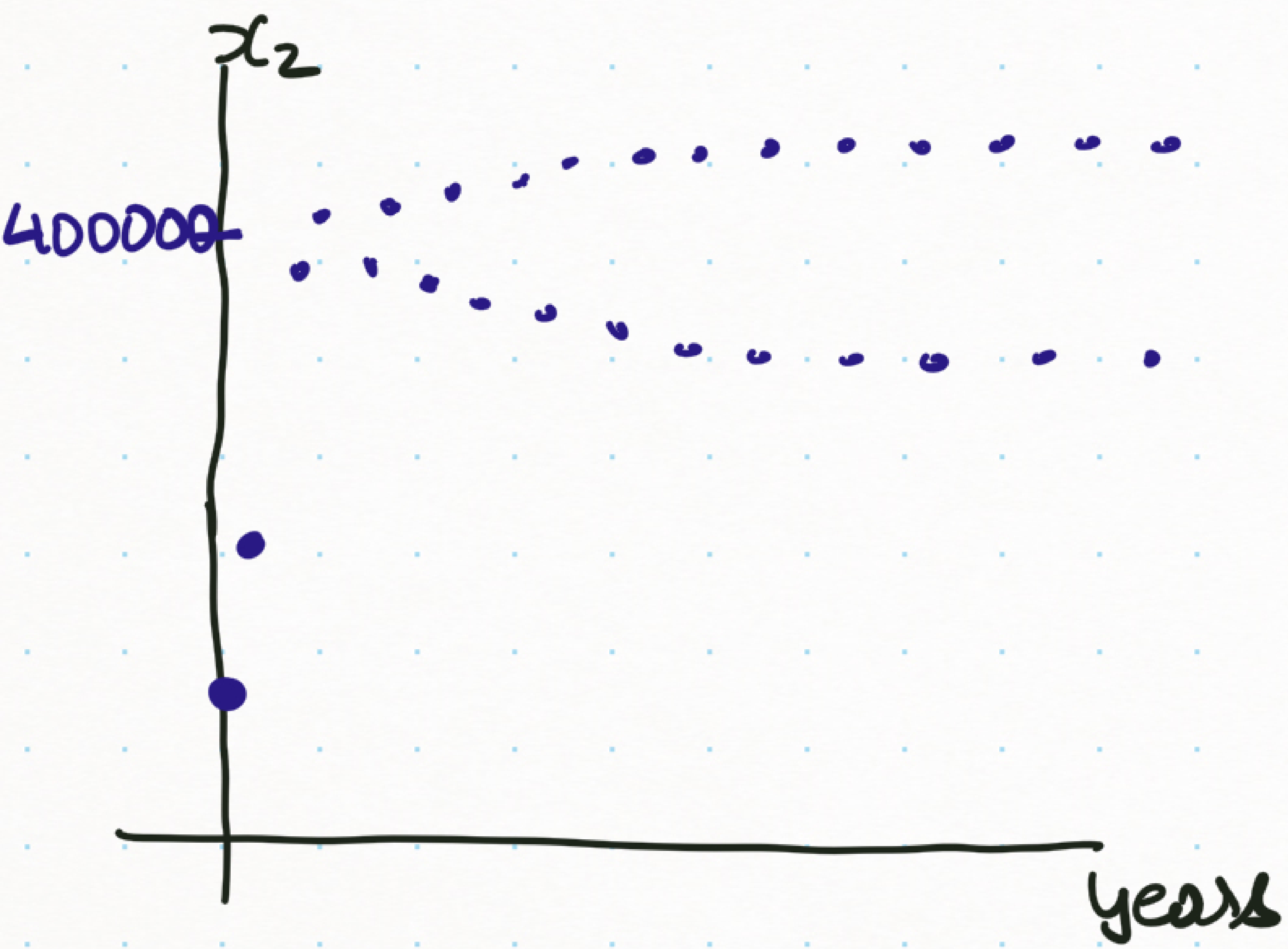
Wouldn't we converge to eq. value faster?



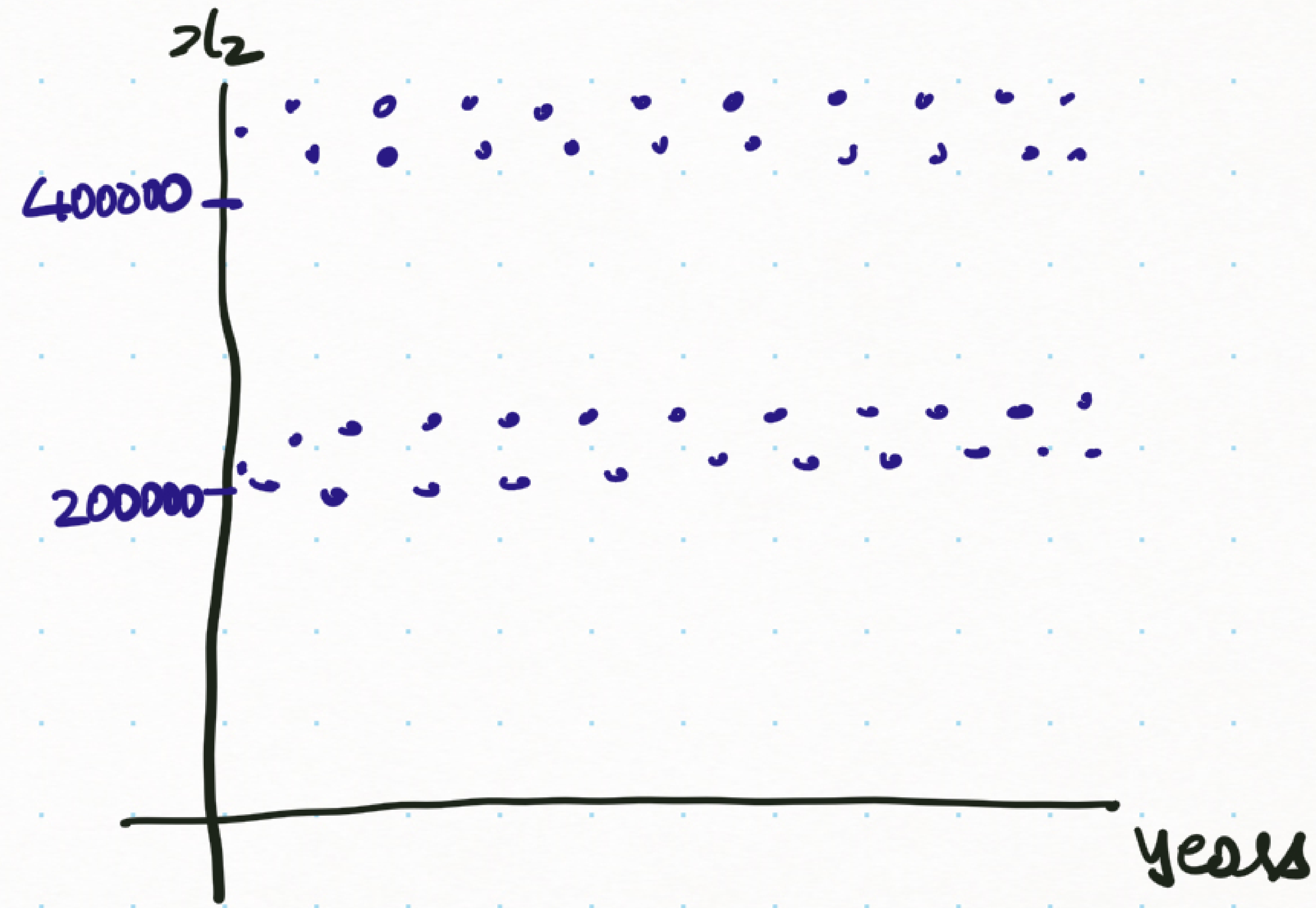
$\Delta t = 2$



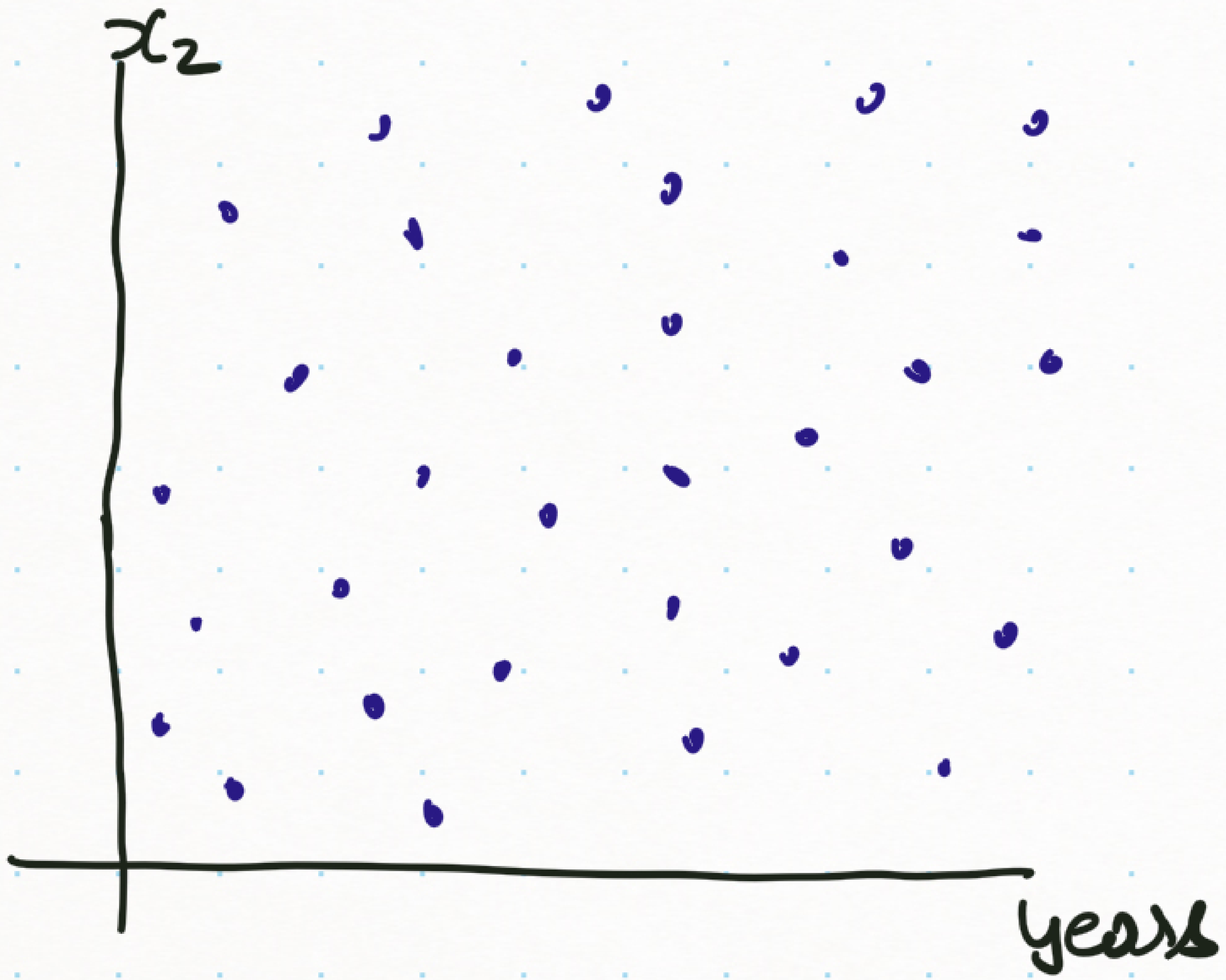
$\Delta t = 24$



$\Delta t = 27$



$\Delta t = 32$



$\Delta t = 37$

Emergence of chaos from discrete approximation

Numerical headache!

But can also emerge naturally from realistic models!

MATH 380

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Continuous Optimization Models

A pig weighing 200 lbs gains 5 lbs/day and costs 45¢/day to keep.

The market price for pigs is 65¢/lb but falling 1¢/day.

When should the pig be sold?

Continuous Optimization Models

A pig weighing 200 lbs gains 5 lbs/day and costs 45¢/day to keep.

The market price for pigs is 65¢/lb but falling 1¢/day.

When should the pig be sold?

An interpretation of

A pig weighs approx 200 lbs.

This pig has been gaining 5 lbs/day for the past week.

Five days ago the pig could have been sold for 70¢/lb but now the price has dropped to 65¢/lb

What should we do?

Continuous Optimization Models

Optimization : maximize or minimize a function of decisionmaking variables subject to some constraints on those variables.

e.g. In our "pig example"

max Profit = Revenue − Cost

Continuous Optimization Models

Optimization: maximize or minimize a function of decisionmaking variables subject to some constraints on those variables.

e.g. In our "pig example"

max Profit = Revenue − Cost

A mathematical model is robust if its conclusions remain more or less the same even if the model (incl. underlying parameters / data) is not completely accurate.

Sensitivity Analysis is a way to gauge robustness with respect to assumptions about the data.

we saw this in the numerical computations w.r.T. α in Blue (Fin whales model.

Continuous Optimization Models

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Continuous Optimization Models

A pig weighing 200 lbs gains 5 lbs/day and costs 45¢/day to keep.

The market price for pigs is 65¢/lb but falling 1¢/day.

When should the pig be sold?

Let t = days we wait before selling the pig

Weight $w(t) = 200 + 5t$. Cost $C(t) = 45t$; price/lbs $p(t) = 65 - t$

Revenue, $R(t) = p(t)\,w(t)$

Profit $P(t) = R(t) - C(t) = (65-t)(200+5t) - 45t$

$$= -5t^2 + 80t + 13000$$

Maximize $P(t)$: $\frac{dP}{dt} = -10t + 80 = 0 \Rightarrow t = 8$ days for profit $P(8) = 13320$¢

$= \$133.20$

Continuous Optimization Models

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Profit $P(t) = R(t) - C(t) = (65-t)(200+5t) - 45t$

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Maximize $P(t)$: $\frac{dP}{dt} = -10t + 80 = 0 \Rightarrow t = 8$ days for profit $P(8) = 13320$¢

$= \$133.20$

Criticism of the model?

Robustness? What are we assuming?

- We can't really sell it at <u>any</u> time t, e.g. 3am, etc.

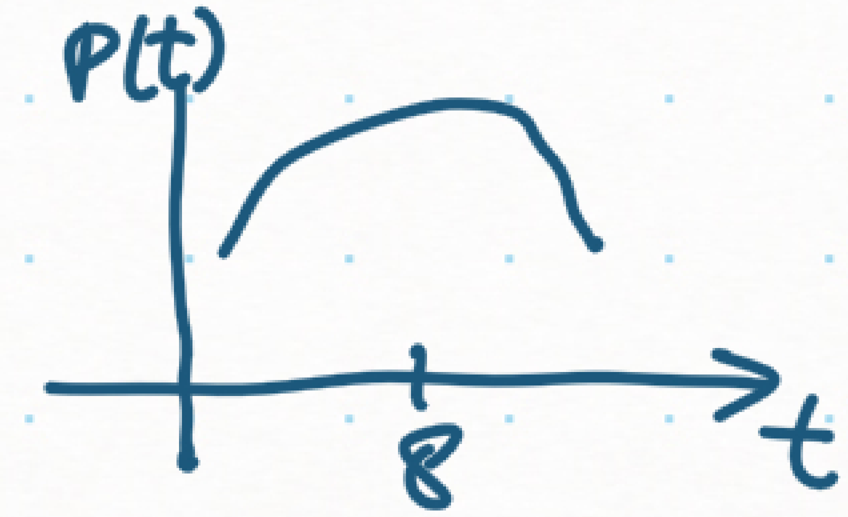
  Does that matter?

  Check nearby values of t. $P(8) = 13320$ $P(7) = 13315$ $P(9) = 13315$

  So, 8 days $\rightarrow$ Profit $= \$133.20$ 7 or 9 days $\rightarrow$ Profit $= \$133.15$

  $$P(t) = -5t^2 + 80t + 1300$$

  is a parabola



Robustness? What are we assuming?

- The previous weight gain of 5lbs/day may not be accurate

Different weight gain assumptions?

Let g be the pounds gained per day

~~If~~ If $g=6$: $w(t)=200+6t$, so $P(t)=(65-t)(200+6t)-45t$

$$\frac{dP}{dt}=0 \iff (65-t)6+(-1)(200+6t)-45=-12t+390-200-45=0$$

i.e., $t=\frac{145}{12}\approx 12$ days

$P(12)=13876$, ie. Profit \$138.76

Robustness? What are we assuming?

- The previous weight gain of 5lbs/day may not be accurate

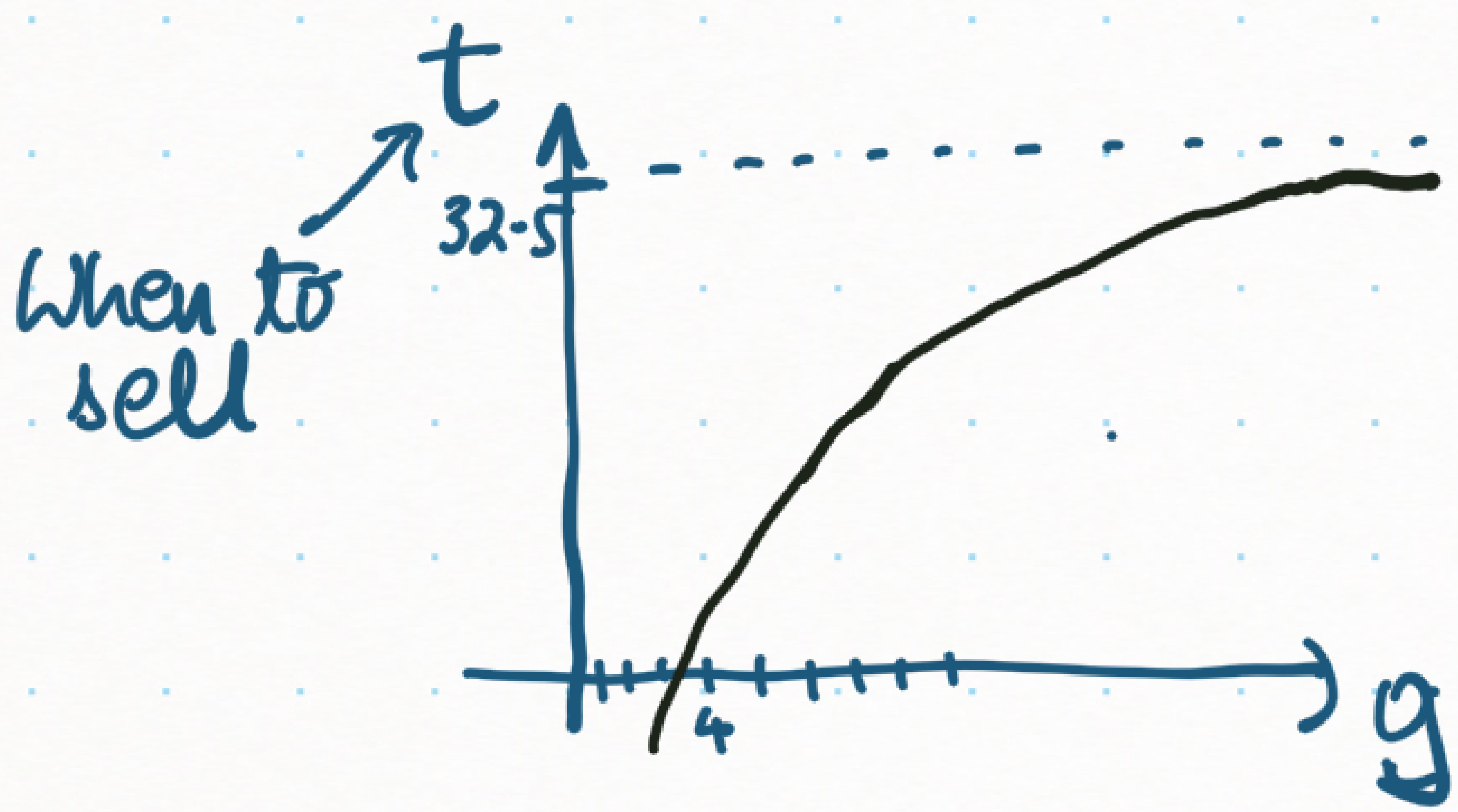
Different weight gain assumptions?

Let g be the pounds gained per day

For any constant g: $w(t) = 200 + gt$, $P(t) = (65-t)(200+gt) - 45t$

$$\frac{dP}{dt} = -(200+gt) + (65-t)g - 45 = -2gt + 65g - 245 = 0$$

i.e., $\boxed{t = \frac{65g - 245}{2g} = 32.5 - 122.5/g}$ Best time to sell



Interpretation?

Robustness? What are we assuming?

- The previous weight gain of 5lbs/day may not be accurate

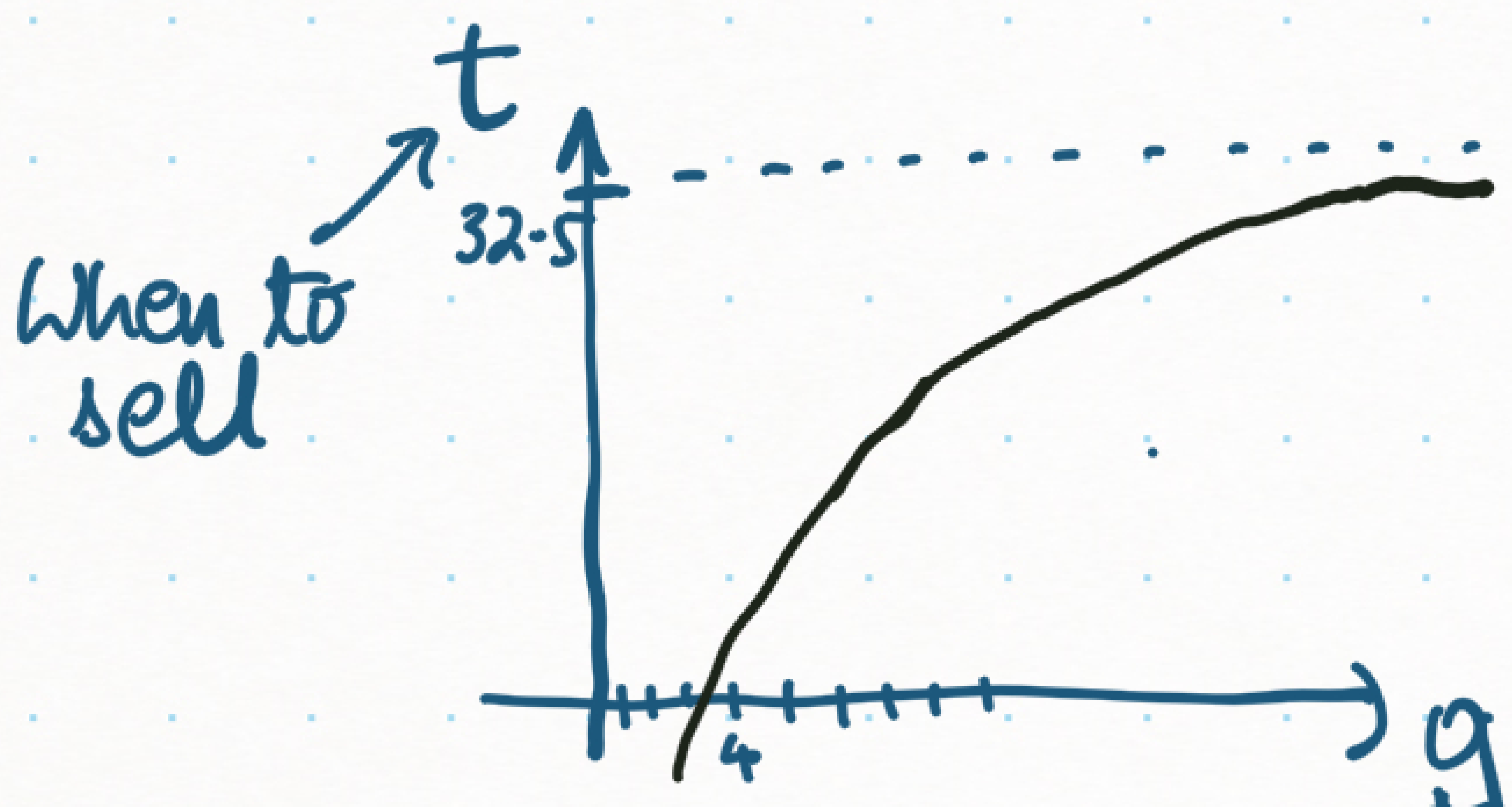
  Different weight gain assumptions?

  Let g be the pounds gained per day

  For any constant g: $w(t) = 200 + gt$, $P(t) = (65-t)(200+gt) - 45t$

  $$\frac{dP}{dt} = -(200+gt) + (65-t)g - 45 = -2gt + 65g - 245 = 0$$

  i.e., $\boxed{t = \frac{65g-245}{2g} = 32.5 - 122.5/g}$ Best time t to sell



Interpretation?

Below $g = 245/65$, t is 0

Above $g = 245/65$, t increasing but only upto 32.5 days

Robustness? What are we assuming?

- The previous weight gain of 5lbs/day may not be accurate

Different weight gain assumptions?

Let g be the pounds gained per day

For any constant g: $w(t) = 200 + gt$, $P(t) = (65-t)(200+gt) - 45t$

$$\frac{dP}{dt} = -(200+gt) + (65-t)g - 45 = -2gt + 65g - 245 = 0$$

i.e., $\boxed{t = \dfrac{65g - 245}{2g} = 32.5 - 122.5/g}$ Best time to sell

So, optimal time to sell is $t^* = \begin{cases} (65g-245)/2g & \text{if } g \geq 245/65 \\ 0 & \text{if } g < 245/65 \end{cases}$

& at that time, Profit is $P(t^*) = \begin{cases} (25/4g)(169g^2 + 806g + 2401) & \text{if } g \geq 245/65 \\ 13000 & \text{if } g < 245/65 \end{cases}$

Inter-pretation: when to sell?

Robustness? What are we assuming?

- The previous weight gain of 5lbs/day may not be accurate

Different weight gain assumptions?

Growth rate is not a constant, but is proportional to its current weight.

$$\frac{dw}{dt} = cw \quad \text{for a constant } c>0.$$

with $w(0)=200$

What is c?

Then $\frac{dw}{w} = c\,dt \Rightarrow w(t) = 200\,e^{ct}$

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with $w(0) = 200$

Then $\frac{dw}{w} = c\,dt \Rightarrow w(t) = 200\, e^{ct}$

What is c?

we can estimate c as $\frac{dw}{dt} \approx 5$ lbs/day when $w = 200$ lbs

so, $c \approx \frac{5}{200} = 0.025$

Profit $P(t) = (65-t)(200e^{ct}) - 45t$

How sensitive is this to value of c?

For any fixed c, the optimum occurs at t for $P'(t) = 0$

$P'(t) = 200c\,e^{ct}(65-t) - 200e^{ct} - 45$

continuous function of c, so not much difference between $c = 0.025$ & values of c nearby.

Robustness? What are we assuming?

- The previous weight gain of 5lbs/day may not be accurate

Different weight gain assumptions?

Growth rate is not a constant, but is proportional to its current weight.

$$\frac{dw}{dt} = cw \quad \text{for a constant } c>0.$$

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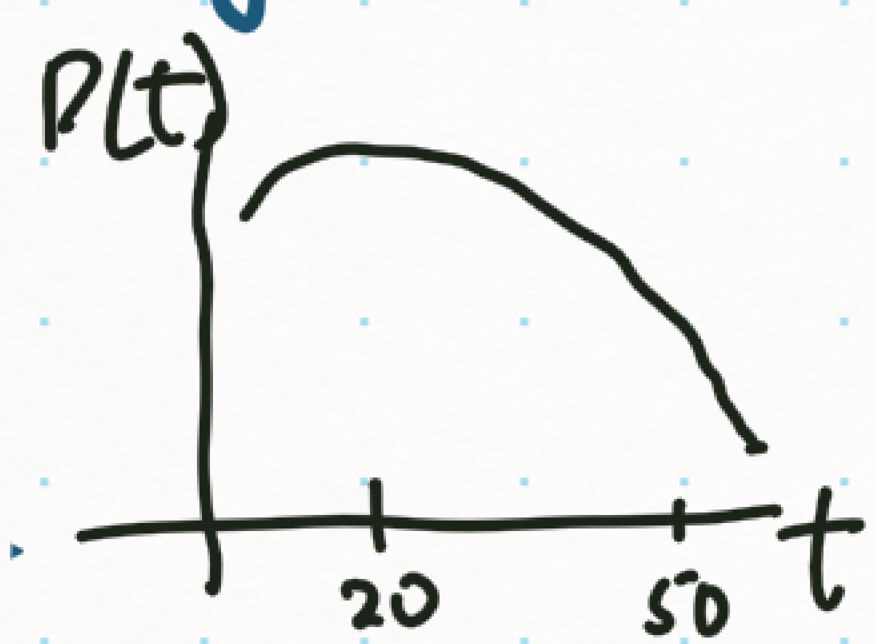
Profit $P(t) = (65-t)(200e^{ct}) - 45t$



with $c = 0.025$

max at $t = 19.5$

with Profit $= \$139.39$

Robustness? What are we assuming?

- Market price may not fall by 1¢/day

  Different market price model?

  Say, market price falls by r ¢/day

  Then, $P(t) = (65 - rt)(200 + 5t) - 45t$ (assuming 5lbs/day again)

Robustness? What are we assuming?

- Market price may not fall by 1¢/day

  Different market price model?

  Say, market price falls by r ¢/day

  Then, $P(t) = (65 - rt)(200 + 5t) - 45t$ (assuming 5lbs/day again)

  $\frac{dP}{dt} = 0 \iff t = \frac{28}{r} - 20$ as best time to sell.

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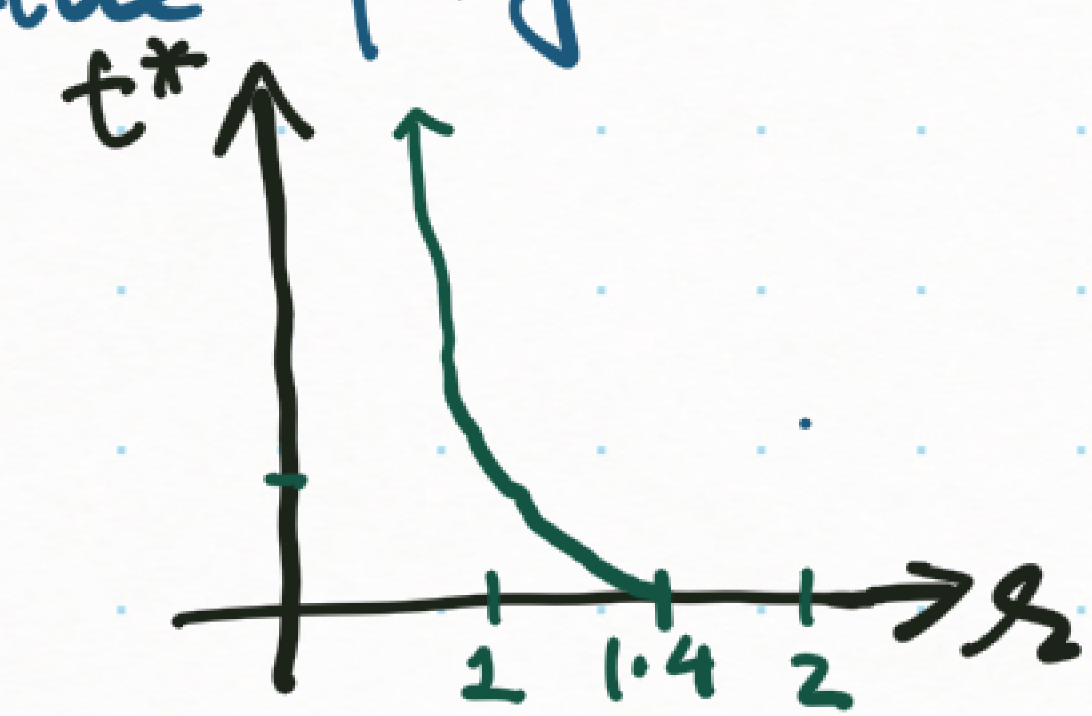
Say, market price falls by r ¢/day

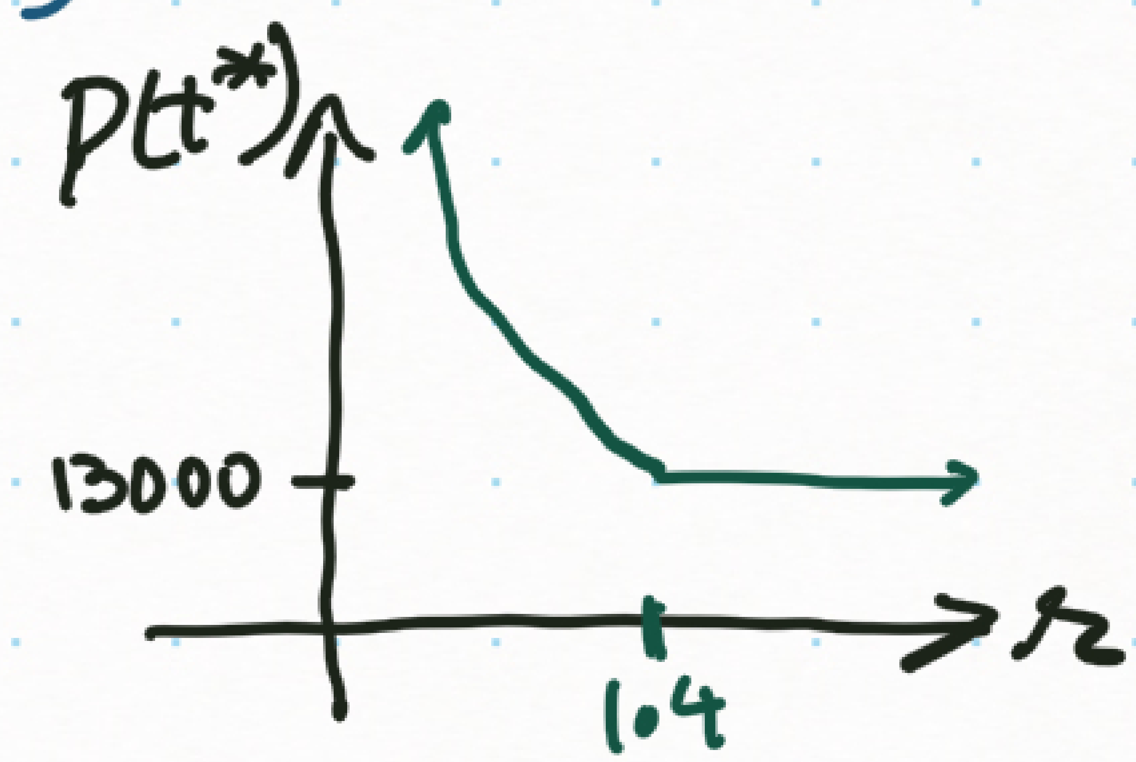
Then, $P(t) = (65 - rt)(200 + 5t) - 45t$ (assuming 5lbs/day again)

$\frac{dP}{dt} = 0 \iff t = \frac{28}{r} - 20$ as best time to sell.

So, if $t^* = \begin{cases} 28/r - 20 & \text{if } r \leq 28/20 = 1.4 \\ 0 & \text{if } r > 28/20 = 1.4 \end{cases}$

and the profit will be $P(t^*) = (40/r)(50r^2 + 185r + 98)$ for $r \leq 1.4$





Sensitivity

The sensitivity of y to x compares relative change in y to relative change in x.

e.g. If x goes up by 2% then y goes down 7% then sensitivity of y to x is $\frac{-7\%}{2\%} = -3.5$

More, generally, $(\Delta y / y) / (\Delta x / x)$

For a continuous model, we consider

$$\lim_{\Delta x, \Delta y \to 0} (\Delta y / y) / (\Delta x / x), \text{ i.e., } \lim_{\Delta x, \Delta y \to 0} \left(\frac{\Delta y}{\Delta x}\right)\left(x/y\right)$$

Definition The sensitivity of y to x is

$$S(x,y) = \left(\frac{dy}{dx}\right) * \left(x/y\right)$$

<u>Recall</u> Profit $P(t) = (65-t)(200+t) - 45t$

we found optimal time, $t^* = 8$ days

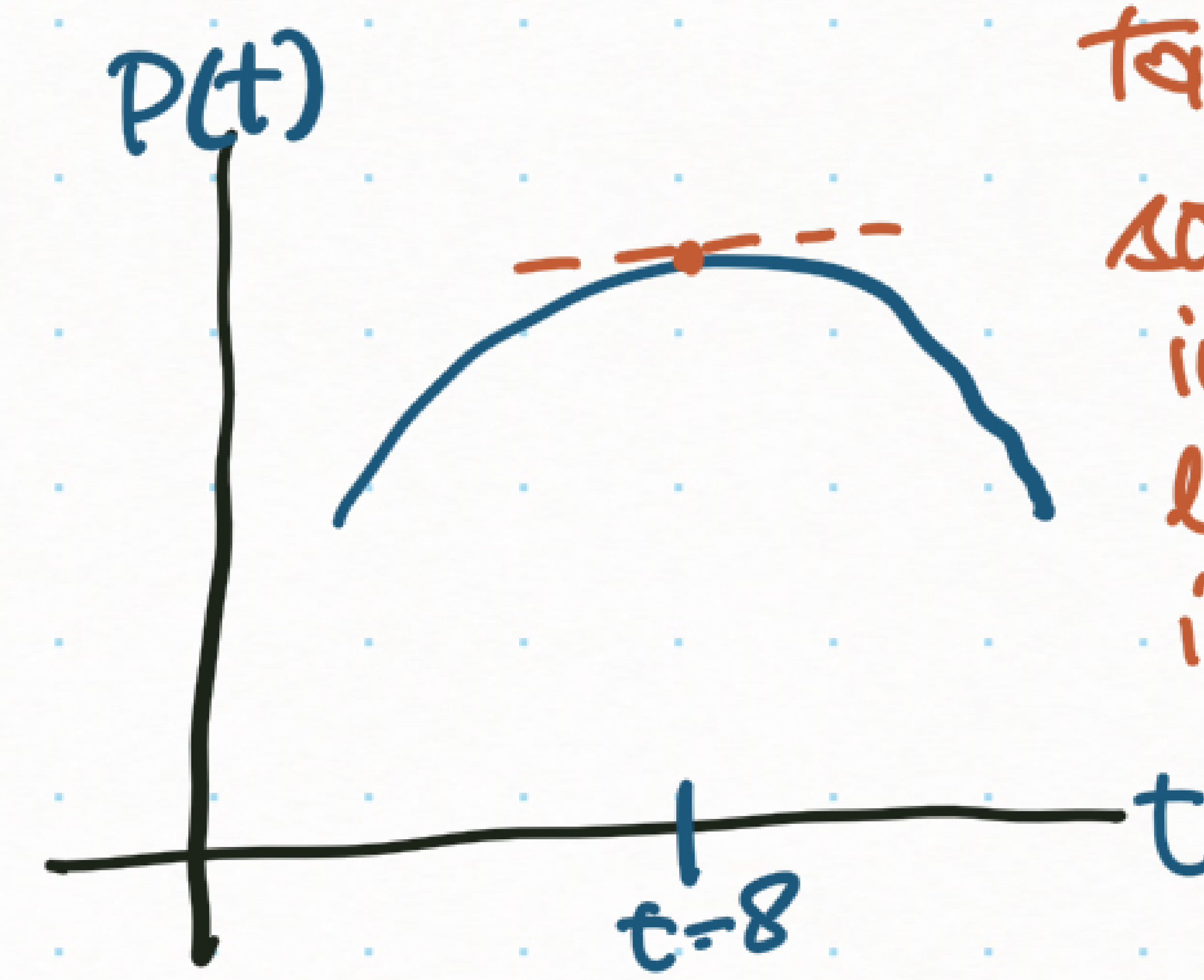
with profit $P(t^*) = \$13.32$

<u>Sensitivity of profit to time</u>:

$$S(P,t) = \left(\frac{dP}{dt}\right)\left(\frac{t}{P}\right) = (-10t+80)\left(\frac{t}{(65-t)(200+t)-45t}\right)$$

$$= 0 \text{ when } t = 8$$

The model is robust, not sensitive to getting the time to sell exactly correct



tangent slope is 0 so small change in t will mean little change in $P(t)$.

Sensitivity to daily weight gain g [when $r = 1$ ¢/day, etc.]

$P(t) = (65-t)(200+gt) - 45t$, where optimal $t = (65g-245)/2g$

Sensitivity $S(t,g) = (dt/dg)(g/t) = \left(\frac{245}{2}\right)g^{-2}$

At $g=5$, $S(t,g) = 4.9$

Interpretation: If g is not exactly 5, then the optimal sell time t will be off by factor of 4.9 w.r.t. relative error of g

e.g. if g is 1% off from 5, then t will be about 4.9% off from 8 days

Sensitivity $S(P,g) = \left(\frac{dP}{dg}\right)(g/P)$ [At optimal selling time $P = (25/4)(169g + 806 + 2401/g)$ if $g \geq \frac{245}{65}$]

$= \left(\left(\frac{25}{4}\right)(169 - 2401/g^2)\right)\left(g/\left(\frac{25}{4}\right)(169g + 806 + \frac{2401}{g})\right)$

$= g(169 - 2401/g^2)/(169g + 806 + 2401/g)$ $\therefore$ at $g=5$, $S(P,g) = \frac{1.9}{11} = 0.17$

Interpretation: If growth g is, say, 10% faster than expected then profit will be about 1.7% more than expected, etc.

Sensitivity to daily market price drop r

Recall $P=(65-rt)(200+5t)-45t$

$t=\frac{28}{r}-20$, optimal selling time if $r\leq 1.4$

for a profit of $P=40(50r+185+98/r)$

<u>Sensitivity $S(t,r)$</u> $=\left(\frac{dt}{dr}\right)\left(\frac{r}{t}\right)=\left(-28/r^2\right) r/(28r-20)$

At $r=1$, $S(t,r)=\frac{-28}{8}=-3.5$ Interpretation?

<u>Sensitivity $S(P,r)$</u> $=\left(\frac{dP}{dr}\right)\left(\frac{r}{P}\right) = (50-98/r^2)\left(r/(50r+185r+98/r)\right)$

At $r=1$, $S(P,r)=-48/333 \approx 0.144$ Interpretation?

<u>Overall advice</u> ?

Sensitivity to daily market price drop r

Recall $P=(65-rt)(200+5t)-45t$

$t=\frac{28}{5}-20$, optimal selling time if $r\leq 1.4$

for a profit of $P=40(50r+185+98/r)$

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At $r=1$, $S(P,r)=-48/333\approx 0.144$ Interpretation?

Overall advice Wait a week or so, and reevaluate the situation.