MATH 400: Homework #9

Only the 'Submission Problems' listed below are due Wednesday, 11/30, before 11:30pm, via a PDF file uploaded to the Homework#9 under Assignments in the Canvas course page.

You are allowed to discuss the homework problems with no one except your classmates, the TA, and the instructor. However, the solutions should be written by you and you alone in your own words. **If you discussed HW problems with a classmate or TA, you have to write their name at the top of the HW submission as a collaborator.** Any incident of plagiarism/ cheating (from a person or from any online resource) will be strictly dealt with.

Re-read the "HW Discussion and Solution Rules" and " 'Why and How' of Homework" sections of the course information sheet for some important advice on the HWs for this course.

All problems require explicit and detailed explanations. Solutions should be written clearly, legibly, and concisely, and will be graded for both mathematical correctness and presentation. Points will be deducted for sloppiness, incoherent or insufficient explanation, or for lack of supporting rationale.

Always remember that homework is NOT meant to be an examination, it is meant to assist in your learning and development. If you need help with any HW problem, don't hesitate to ask me. You are encouraged to ask questions through the *Canvas Discussion Forums*, during my *Office Hours*, or through *Email to me*.

PART I: Practice Problems

1. Try all the Discussion/ Review problems as you review the material for this week.
2. Make your own summary of this week's topics/ concepts/ properties: definitions, alternate forms of the definition based on theorems/ discussions, examples and non-examples, methods for showing the property holds or doesn't hold, useful ideas from examples, HW problems, etc.
3. Pay close attention to and attempt at least the following exercises from the textbook.
   Section 4.4: #1, #2, #3, #5, #7, #9 (make a note of Lipschitz functions and their uniform continuity), #10, #11 (Note this topological definition of continuity), #13 (Note this very useful technique, which I partly illustrate in Discussion Question #7).
   Section 5.2: #1, #3, #5, #7, #9, #11 (Be sure to finish this discussion from the lecture), #12 (Make a note of this Inverse Function Derivative Rule).

PART II: Submission Problems

4. **Submit written solutions to the problems listed below.**
   - Section 4.4: #1, #7, #9.
   - Section 5.2: #3, #7.

PART III: Readings, Comments, etc.

5. You have already done many many (two semesters worth) of examples applying rules and properties of differentiation to find the derivatives and max/min of complicated functions in your study of single-variable calculus. In the context of our course, its more important and meaningful for us to think about how the fundamental notions of continuity, differentiability, etc. are related to each other.

   We have discussed functions that are counterexamples to the following false statements:

   - f is continuous at a implies f is differentiable at a.
   - f is differentiable implies its derivative is continuous.

   Can you construct functions that are counterexamples to the following false statements?

   - f is not differentiable at a implies f^2 is not differentiable at a.
   - f is not differentiable at any point in a set A implies f^2 is not differentiable at every point in A.
   - f is not differentiable at a implies $|f|$ is not differentiable at a. (Recall we already know a function such that f is differentiable at a but $|f|$ is not differentiable at a. So there is no direct relation between differentiability of f vs. $|f|$.)
   - If f is infinitely differentiable on $\mathbb{R}$, i.e., $f^{(n)}$ the nth order derivative exists for each n, and $f(0) = f^{(n)}(0) = 0$ for all n, then $f(x) = 0$ for all $x \in \mathbb{R}$.
   - If $\lim\limits_{h\to\infty} \frac{f(x+h)-f(x-h)}{2h}$ exists, then $f'(x)$ exists.
   - $h(x) = f(x) + g(x)$ is differentiable at a implies both $f(x)$ and $g(x)$ are differentiable at a.
   - $h(x) = f(x)g(x)$ is differentiable at a implies both $f(x)$ and $g(x)$ are differentiable at a.
   - $h(x) = f(x)/g(x)$ is differentiable at a implies both $f(x)$ and $g(x)$ are differentiable at a.
   - If $h(x) = f(x)g(x)$ is differentiable at a and $f(x)$ is differentiable at a, then $g(x)$ must be differentiable at a.
   - If $f(x)$ is not differentiable at a and $g(x)$ is differentiable at $f(a)$, then $g(f(x))$ is not differentiable at a.
   - If $f(x)$ is differentiable at a and $g(x)$ is not differentiable at $f(a)$, then $g(f(x))$ is not differentiable at a.
   - If $f(x)$ is not differentiable at a and $g(x)$ is not differentiable at $f(a)$, then $g(f(x))$ is not differentiable at a.
   - There is no function that is differentiable only at a single point of its domain.
   - If f is differentiable at a then f is continuous in some neighborhood of a.

6. If you enjoy thinking about and playing with 'weird' functions, ask me for recommendations for books that focus on examples and counterexamples of the sort listed above.