MATH 400: Homework #11

Only the 'Submission Problems' listed below are due Wednesday, 11/20, before 11:30pm, via a PDF file uploaded to the Homework#11 under Assignments in the Canvas course page.

You are allowed to discuss the homework problems with no one except your classmates, the TA, and the instructor. However, the solutions should be written by you and you alone in your own words. **If you discussed HW problems with a classmate or TA, you have to write their name at the top of the HW submission as a collaborator.** Any incident of plagiarism/ cheating (from a person or from any online resource) will be strictly dealt with.

Re-read the "HW Discussion and Solution Rules" and " 'Why and How' of Homework" sections of the course information sheet for some important advice on the HWs for this course.

All problems require explicit and detailed explanations. Solutions should be written clearly, legibly, and concisely, and will be graded for both mathematical correctness and presentation. Points will be deducted for sloppiness, incoherent or insufficient explanation, or for lack of supporting rationale.

Always remember that homework is NOT meant to be an examination, it is meant to assist in your learning and development. If you need help with any HW problem, don't hesitate to ask me. You are encouraged to ask questions through the *Canvas Discussion Forums*, during my *Office Hours*, or through *Email to me*.

PART I: Practice Problems

1. Try all the Discussion/ Review problems as you review the material for this week.

2. Make your own summary of this week's topics/ concepts/ properties: definitions, alternate forms of the definition based on theorems/ discussions, examples and non-examples, methods for showing the property holds or doesn't hold, useful ideas from examples, HW problems, etc.

3. Pay close attention to and attempt at least the following exercises from the textbook.
   Section 6.4: #1, #3, #5, #7, #9.
   Section 6.5: #1 (be sure to try this), #3, #5, #7 (for part (c), all you need is a stronger form of Ratio test to make the arguments in parts (a) and (b) work for part (c) as well. Review the solution for Exercise 2.7.9 from HW#5 to see how this stronger form of Ratio test, based on the given condition, would be proved.).
   Section 6.6: #1 (notice the connection to $\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}$ equals $\frac{\pi}{4}$.), #2 (be sure to try this), #3, #4 (after talking about it in Chapter 2, we are finally ready to do this!), #5, #7, #9 (make a note of this exercise for the formal definition of Taylor series centered at a that is not necessarily 0 and the corresponding Cauchy remainder formula.).
   Section 7.2: #1, #2, #3, #5 (be sure to try this and note the importance of uniform convergence here), #6 (note Riemann's original definition of his integral here. What we use is originally Darboux' definition of integral, but they are equivalent definitions.), #7 (this problem is a simple but good review of the basic definitions, try it.).
   Section 7.3: #1, #3, #5, #7.

PART II: Submission Problems

4. **Submit written solutions to any five (including the Special Problem A) of the six problems listed below.**

   - Section 6.4: #5.
   - Section 6.5: #7ab and then apply it to find the exact interval of convergence for the power series $\sum_{n=0}^{\infty}(\frac{3^n}{n4^n})x^n$.
   - Section 6.6: #4 (after talking about it in Chapter 2, we are finally ready to do this!), #7 (review the counterexample on page 203 before attempting this problem).
   - Section 7.2: #3.
   - [Special Problem A]
     Let $f : [0,1] \to \mathbb{R}$ be defined as $f(x)$ equals 0 at rationals in $[0,1]$ and x at irrationals in $[0,1]$. Show that f is not integrable on $[0,1]$.

PART III: Readings, Comments, etc.

5. Coming some day soon - A discussion of Lebesgue integral.