Math 400: Discussion Questions/ Review # 7

A statement listed with [T/F] is a True/False statement that requires a proof or a counterexample, as appropriate.

1. Is $\{3\}$ compact?
2. Is the Cantor set compact?
3. Is $\{x \in \mathbb{R} : 2 < |x| < 4\}$ compact?
4. Is $\{x \in \mathbb{R} : 2 \leq |x| \leq 4\}$ compact?
5. Is $\{-1, 2, \pi, 7, -16\}$ compact? Give two different proofs for your answer.
6. Is $\mathbb{R}$ compact? Give two different proofs for your answer.
7. Consider the set $(0, 4)$ and the collection $\mathcal{U} = \{(\frac{1}{k}, 4 - \frac{1}{k})\}_{k=1}^{\infty}$.
   (a) Verify that $\mathcal{U}$ is an open cover of $(0, 4)$.
   (b) Verify that $\mathcal{U}$ does not have a finite subcover of $(0, 4)$.
   (c) What does this tell about $(0, 4)$?
8. [T/F] If $f(x) = 4x + 8$, then $\lim_{x \to 3} f(x) = 20$.
9. Complete the proof of $\lim_{x \to 2} x^2 = 4$.
10. [T/F] If $f(x) = 3x - 2$, then $\lim_{x \to 4} f(x) = 20$.
11. [T/F] $\lim_{x \to 0} \sin(\frac{1}{x}) = 0$
12. [T/F] $\lim_{x \to 0} \sin(\frac{1}{x}) = 1$