Math 400: Discussion Questions/ Review # 6

A statement listed with [T/F] is a True/False statement that requires a proof or a counterexample, as appropriate.

1. Review for Cantor Set:

   (a) How many intervals are in the stage 5 of the Cantor set? What is the length of each of those intervals?

   (b) What is the length of the Cantor set? Justification?

   (c) What is the cardinality of the Cantor set? Justification?

   (d) What is the dimension of the Cantor set? Justification?

   (e) What is the definition of Sierpinski set? the definition of Menger cube?

2. Is the empty set open? Is $\mathbb{R}$ open?
3. Is $\mathbb{R} \setminus \{5\}$ open?
4. Is $\{5\}$ open?
5. Is $\mathbb{Q}$ open?
6. Is the interval $(2, 5)$ open?
7. Is the interval $[2, 5]$ open?
8. Is the interval $(2, 5]$ open?
9. Is the interval (a, ∞) open?
10. Is the interval $(-\infty, b)$ open?
11. [T/F] Union of countably many open sets is an open set.
12. [T/F] Union of uncountably many open sets is an open set.
13. [T/F] Intersection of finitely many open sets is an open set.
14. [T/F] Intersection of countably many open sets is an open set.

15. Is the empty set closed? Is $\mathbb{R}$ closed?
16. Is $\mathbb{R} \setminus \{5\}$ closed?
17. Is $\{5\}$ closed?
18. Is $\mathbb{Q}$ closed?
19. Is the interval $(2, 5)$ closed?
20. Is the interval $[2, 5]$ closed?
21. Is the interval $(2, 5]$ closed?

22. Is the interval (a, ∞) closed?
23. Is the interval $(-\infty, b)$ closed?

24. Give an example of a set that both open and closed.
25. Give an example of a set that neither open nor closed.

26. [T/F] Union of finitely many closed sets is a closed set.
27. [T/F] Union of countably many closed sets is a closed set.
28. [T/F] Intersection of countably many closed sets is a closed set.
29. [T/F] Intersection of uncountably many closed sets is a closed set.

30. [T/F] If a is a limit point of A, then $a \in A$.
31. [T/F] a is a limit point of A iff there is a sequence $(a_n) \subset A$ with $a_n \to a$.
32. [T/F] If a set is closed then it contains all its limit points.
33. [T/F] If a set contains all its limit points then its closed.
34. [T/F] There exists an open set that contains all its limit points.
35. [T/F] If a set contains all its limit points then its not open.

36. What is the closure of $\{5\}$?
37. What is the closure of $\{5, 6\}$?
38. What is the closure of $(2, 4)$?
39. What is the closure of $(3, 5]$?
40. What is the closure of $\mathbb{Q}$?
41. What is the closure of $\mathbb{R}$?
42. [T/F] $A \subseteq \overline{A}$.
43. [T/F] $\overline{A} \subseteq A$.
44. [T/F] Closure$(\overline{A}) = \overline{A}$.