Math 400: Discussion Questions/ Review # 11

A statement listed with [T/F] is a True/False statement that requires a proof or a counterexample, as appropriate.

1. [T/F] $\sum_{n=1}^{\infty} \frac{\cos nx}{n^2}$ is uniformly convergent on $\mathbb{R}$.
2. [T/F] $\sum_{n=1}^{\infty} \frac{\sin nx}{n!}$ is uniformly convergent on $\mathbb{R}$.
3. [T/F] $\sum_{n=1}^{\infty} x^n \sin nx$ is uniformly convergent on $[-\frac{1}{2}, \frac{1}{2}]$.
4. [T/F] $\sum_{n=1}^{\infty} \frac{\cos nx}{n\sqrt{n+1}}$ is a continuous function on $\mathbb{R}$.
5. Find an interval such that $\sum_{n=1}^{\infty} \frac{x^{2n}}{n2^n}$ is a continuous function on that interval.
6. [T/F] $\frac{1}{1-x^3} = 1 + x^3 + x^6 + x^9 + \dots$, for all $x \in (-1, 1)$.
7. [T/F] $\frac{3x^2}{(1-x^3)^2} = 3x^2 + 6x^5 + 9x^8 + \dots$, for all $x \in (-1, 1)$.
8. [T/F] $\frac{1}{1+x^2} = 1 - x^2 + x^4 - x^6 + x^8 - \dots$, for all $x \in (-1, 1)$.
9. [T/F] $\arctan(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9} - \dots$, for all $x \in (-1, 1)$.
10. [T/F] $\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \dots$, for all x.
11. [T/F] $\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \dots$, for all x.
12. [T/F] $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$, for all x.
13. Let $i = \sqrt{-1}$. Show that $e^{ix} = \cos(x) + i\sin(x)$. Use this to conclude $e^{i\pi} + 1 = 0$.
14. [T/F] Every continuous function is integrable.
15. [T/F] If there exists a partition P of $[a, b]$, such that $L(f, P) = U(f, P)$, then f is integrable over $[a, b]$.
16. Use the definition of Riemann integral to find the value of the integral of $f(x) = x^2$ over the interval $[0, 1]$. (You might find the formula: $1^1 + 2^2 + \dots + n^2 = \frac{1}{6}n(n+1)(2n+1)$ useful.)
17. [T/F] Dirichlet Function is integrable on $[0, 1]$.
18. [T/F] Let $f : [0, 2] \to \mathbb{R}$ be defined as $f(x) = 1$ at all points except 1 and 1.5 where it is 0. Then, f is integrable on $[0, 2]$.
19. Let f be a function defined on $[a, b]$. Suppose f has k points of discontinuities at $c_1 < c_2 < \dots < c_k$ in $[a, b]$. Define P_ϵ, the partition of $[a, b]$, that can be used to verify the integrability criterion for f on $[a, b]$.
    Is there any other method we can use for showing f is integrable on $[a, b]$?
20. [T/F] Let $f : [a, b] \to \mathbb{R}$ be defined as $f(x)$ equals 1 at rationals in $[a, b]$ and 0 at irrationals in $[a, b]$. Then f is integrable over $[a, b]$.

21. Give an example for: a sequence of integrable functions whose pointwise limit function is not integrable.

22. [HW?] Let $f : [0, 1] \to \mathbb{R}$ be defined as $f(x)$ equals 0 at rationals in $[0, 1]$ and x at irrationals in $[0, 1]$. Show that f is not integrable on $[0, 1]$.