Math 590 — Computer Assignment 2 — due Wednesday, Oct. 8, 2014

1. (a) Write a function `TPKernelMatrix.m` that takes two sets of points (data sites and centers) as well as a 1D kernel K as input and returns the kernel matrix K associated with a d-dimensional tensor product kernel.

   (b) Test your function with the kernel $K(\boldsymbol{x}, \boldsymbol{z}) = \prod_{\ell=1}^{d} (\min\{x_\ell, z_\ell\} - x_\ell z_\ell)$ and produce plots for the cases $d = 1$ and $d = 2$. In 1D your plot should show 9 copies of the kernel centered at equally spaced points in $[0, 1]$. In 2D, produce two plots that are analogous to those on p.27 of the slides for Chapter 3.

2. Create a script that computes and plots (in 2D) the optimal kriging weights $\overset{*}{\boldsymbol{w}}(\boldsymbol{x}) = \mathsf{K}^{-1}\boldsymbol{k}(\boldsymbol{x})$ (also known as the cardinal basis functions) for three different kernels of your choice. For each of the kernel you pick, produce plots at 3 different points selected from

   (a) 15×15 evenly spaced points,

   (b) 225 Halton points,

   (c) 225 Sobol′ points.

   Use 40×40 evenly spaced evaluation points for your plots.

3. Create plots of the kriging variance in $[0, 1]^2$ for the Gaussian kernel with $\varepsilon = 5$ for

   (a) 15×15 evenly spaced points,

   (b) 225 Halton points,

   (c) 225 Sobol′ points.

   Use 40×40 evenly spaced evaluation points for your plots.

   Repeat for the Matérn kernels with $\beta = (d + 1)/2$ and $\beta = (d + 5)/2$.

   What do you observe? How do the plots differ for different kernels and different point sets? How are they similar?