1. Determine SVDs of the following matrices. Do not use a computer, and do not use the method for hand calculations discussed in class. Use only basic properties of the SVD and note that the matrices are either diagonal matrices or rank-1 matrices:

   (a) \[
   \begin{bmatrix}
   3 & 0 \\
   0 & -2 
   \end{bmatrix}
   \],
   (b) \[
   \begin{bmatrix}
   2 & 0 \\
   0 & 3 
   \end{bmatrix}
   \],
   (c) \[
   \begin{bmatrix}
   0 & 2 \\
   0 & 0 \\
   0 & 0 
   \end{bmatrix}
   \],
   (d) \[
   \begin{bmatrix}
   1 & 1 \\
   0 & 0 
   \end{bmatrix}
   \],
   (e) \[
   \begin{bmatrix}
   1 & 1 \\
   1 & 1 
   \end{bmatrix}
   \].

2. In the discussion of matrix norms we claimed that the 2-norm of the matrix

   \[
   A = \begin{bmatrix}
   1 & 1 \\
   0 & 1 
   \end{bmatrix}
   \]

   is approximately 1.6180. Using the SVD, work out (the “by-hand” method is from now on allowed) the exact values of \( \sigma_{\min}(A) \) and \( \sigma_{\max}(A) \) for this matrix.

3. Find the SVDs of the following matrices:

   \[
   A = \begin{bmatrix}
   4 & 0 & 0 \\
   0 & 0 & 0 \\
   0 & 0 & 7 \\
   0 & 0 & 0 
   \end{bmatrix},
   B = \begin{bmatrix}
   2 & 1 
   \end{bmatrix},
   C = \begin{bmatrix}
   5 & -4 
   \end{bmatrix}
   .
   \]

4. If \( P \) is an orthogonal projector, then \( I - 2P \) is unitary. Prove this algebraically, and give a geometric interpretation.

5. Consider the matrices

   \[
   A = \begin{bmatrix}
   1 & 0 \\
   0 & 1 \\
   1 & 0 
   \end{bmatrix},
   B = \begin{bmatrix}
   1 & 2 \\
   0 & 1 \\
   1 & 0 
   \end{bmatrix}
   .
   \]

   Answer the following questions by hand calculation.

   (a) What is the orthogonal projector \( P \) onto range\( (A) \), and what is the image under \( P \) of the vector \([1, 2, 3]^*\)?

   (b) Same questions for \( B \).