

“Black-box” Complementarity problems – Applications and Algorithms

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Abstract

- Some classical problems in numerical optimization:
 - Unconstrained optimization $\min f(x)$
 - System of nonlinear equations $F(x) = 0$
 - Nonlinear complementarity problems $0 \leq x \perp F(x) \geq 0$.
- In practical problems, $f(x)$ and/or $F(x)$ have no explicit form, we can only observe/evaluate the function values for given x .
- The costs of observation/evaluations may be expensive.

‘Black-box’ Complementarity Problems: Find y such that

$$0 \leq y \perp F(x(y)) \geq 0.$$

where $x(y)$ is the solution of an ‘oracle’ problem dependent on the parameter y and $F(x(y))$ does not have any explicit forms. Such problems arise in many fields of management sciences, network economics, transportation research and regional science. For example, mitigating traffic congestion via charge, imposing resource tax to preserve resources, and establishing a rational social welfare system, etc.

Since only the function value can be observed and/or evaluated for any given variable y , the practical method should be a direct method which only use the function evaluations. In addition, because the function evaluation or observation may be via a costly experiment or a social investigation in practice, it is meaningful to study how reduce the number of function evaluations in the direct methods.