

Nonsymmetric Positive Solutions for Symmetric Dirichlet Elliptic Problems

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Abstract

In this talk the speaker will present some results on the existence of non-radially symmetric positive solutions of the following radially symmetric problems

$$(1) \quad \begin{cases} -\Delta u + (\lambda - h(x))u = (1 - f(x))u^p, & \text{in } \mathbb{R}^N, \\ u > 0, & \text{in } \mathbb{R}^N, \\ u \in H^1(\mathbb{R}^N), \end{cases}$$

where $h(x)$ and $f(x)$ are nonnegative radially symmetric functions in $L^\infty(\mathbb{R}^N)$, $h(x)$ and $f(x)$ have compact support in \mathbb{R}^N , $f(x) \leq 1$ for all $x \in \mathbb{R}^N$, $1 < p < +\infty$ for $N = 1, 2$, $1 < p < \frac{N+2}{N-2}$ for $N \geq 3$, λ is a positive parameter.

When $N = 1, \sigma = 1$, $h(x) = \chi_{[-d, d]}$, $f(x) = \chi_{[-d, d]}$, where $\chi_{[-d, d]}$ stands for the characteristic function of $[-d, d]$, $d > 0$ is a given number), problem (1) is a model in nonlinear optics arising in the study of the asymmetric guided waves in a symmetric structure consisting of three layers of dielectric materials whose refractive indices depend on the density of the electric field.

When $N = 2$, problem (1) represents a convenient scalar approximation of a model arising from the study of the propagation of a monochromatic electric field in optical cylindrical waveguides. Such scalar approximation can be found in engineering and mathematical literature. A mathematical description of the propagation of electromagnetic fields in optical cylindrical waveguides having a defocusing or self-focusing dielectric response can be found in papers of O. John and C. Stuart and C. Stuart.