Radial basis function theory and applications.

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A radial basis function (RBF) s is a function of the form

$$s(\bullet) = p(\bullet) + \sum_{i=1}^{N} \lambda_i \Phi(\bullet - x_i)$$

where p is a low degree polynomial and Φ is some (usually radial) function. The energy minimisation characterisations of an important class of RBFs, called polyharmonic splines, make them ideally suited to many scattered data fitting problems. At first glance these splines appear to be of little practical use as fitting them appears to involve $\mathcal{O}(N^3)$, work, and each subsequent evaluation appears to involve a further $\mathcal{O}(N)$ work. However, it is now well established that these computational costs can be reduced by orders of magnitude and thus RBFs and piecewise RBFs are a practical tool for scattered data fitting problems.

One problem to which RBFs have been successfully applied is the fitting of surfaces to point clouds. This problem arises in many applications such as reverse engineering, animations for the movie industry, and in the custom manufacture of medical prostheses. Other problems to which RBFs have been applied include geological modelling, the numerical solution of PDEs, and modeling fluid flow, for example tidal flow in a harbour.

The first part of the talk will outline some of the basic properties of RBFs and some of the characteristics of these recent applications. Another part will outline some of the approximation theory and numerical analysis which, in combination, make large scale computations with RBFs tractable. Throughout the emphasis will be on illustrating concepts through animations and videos rather than exploring the underlying mathematics in full detail.