
MIGHTY XLVII

November 8, 2008 Illinois Institute of Technology

All talks are in the Life Sciences Building, IIT Main Campus.

Morning Schedule

7:45-12:00	Registration		
8:00- 8:30	Coffee		
8:30- 8:40	Opening Remarks, R. Russell Betts, Dean of College of Science and Letters, Room 111		
	Invited Talk, Room 111		
8:40- 9:40	Douglas B. West <i>Degree Ramsey and On-Line Degree Ramsey Numbers</i>		
9:40-10:00	Coffee Break		
	Room 121	Room 111	Room 240
10:00-10:20	Bridget Tenner <i>The boolean number of a graph</i>	Tao Jiang <i>Compact topological cliques in sparse graphs</i>	Garry Johns <i>A Survey of Iterated Distance Graphs</i>
10:20-10:40	William Vautaw <i>The Circle Space: Zero-Weighted Cycles in a Weighted, Undirected Graph</i>	Mohit Kumbhat <i>Choosability with separation in complete graphs</i>	Matt Walsh <i>The Paranoid Watchman: a search problem on graphs</i>
10:40-11:00	Nart Shawash <i>Connectivity properties and orientation of generalized Cayley graphs generated by transpositions</i>	David Galvin <i>Is the independent set sequence of the hypercube unimodal?</i>	Paul Wenger <i>The Partial Acquisition Number of Graphs</i>
	Break		
	Room 121	Room 111	Room 240
11:10-11:30	Jian Shen <i>A generalization of the Friendship Theorem</i>	Terry McKee <i>Graphs with complete minimal k-vertex separators</i>	Darin Johnson <i>On the Independent Domination Number of a Random Graph</i>
11:30-11:50	Jeong Ok Choi <i>A decomposition problem of regular hypergraphs</i>	Lou Ibarra <i>The clique-separator graph of a chordal graph</i>	Suil O <i>Balloons, Cut-edges, Matchings, and Total Domination number in Regular Graphs of Odd Degree</i>
11:50-12:10	Kevin Milans <i>Cycle Spectra of Hamiltonian Graphs</i>	Michael D. Barrus <i>The A_4-structure of a graph</i>	Christopher Stocker <i>A bound on the domination number of connected cubic graphs</i>
12:10- 2:10	Lunch (Furama)		

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Afternoon Schedule

Invited Talk, Room 111			
2:10- 3:10	László Babai <i>Symmetry, Hadwiger number, pointwise limit of graphs</i>		
3:10- 3:30	Coffee Break		
	Room 121	Room 111	Room 152
3:30- 3:50	Hao Li <i>Group Colorability of Multi-graphs</i>	Yanting Liang <i>On s-Hamiltonian-connected line graphs</i>	Andrew Lyons <i>Complexity of Optimal Accumulation of Partial Derivatives on DAGs</i>
3:50- 4:10	Christine Cheng <i>Counting as an efficient method for computing the distinguishing and distinguishing chromatic numbers of graphs</i>	Yehong Shao <i>Connectivity and Minimum Degree of Iterated Line Graphs</i>	Gregory Puleo <i>Environmental Evolutionary Graph Theory</i>
4:10- 4:30	W. Doug Weakley <i>Fixing sets and resolving sets for the rook's graph</i>	Huiya Yan <i>Bounded number of components of 2-factors in line graphs</i>	Glenn Harris <i>Spinor Components and Graph Theory</i>
	Break		
	Room 121	Room 111	Room 152
4:40- 5:00	Peter Hamburger <i>Destroying cycles in digraphs</i>	John Maharry <i>Unavoidable Minor Structures in Large Graphs</i>	Mustafa Atici <i>$(n, n + k)$-Graphs that maximize the Integrity for small k</i>
5:00- 5:20	Robert (Chip) Vandell <i>Connected Decycling</i>	John Steinberger <i>An unavoidable set of D-reducible configurations</i>	Andrew M Schwartz <i>H-Matchable Trees: Degree and Distance Distribution</i>
5:20- 5:40		Despina Stasi <i>A criterion for embeddability in the projective plane</i>	Eddie Cheng <i>A generating function approach to the surface area of some interconnection networks.</i>
5:40- 6:00		Josh Lambert <i>Finding a Biplanar Imbedding of $C_n \times C_n \times C_l \times P_m$</i>	Le Anh Vinh <i>Finite Euclidean and non-Euclidean graphs</i>

Invited Talks

Symmetry, Hadwiger number, pointwise limit of graphs

László Babai, University of Chicago

Abstract: We discuss a pointwise limit concept for rooted graphs. This concept permits the study of the asymptotic local structure of families of finite graphs of bounded degree via the study of the infinite graphs arising as a limit. Global arguments for finite graphs, such as sphere packings, are then complemented by local arguments using concepts available for infinite graphs, such as their ends.

This method has been particularly successful in the study of the relation between the automorphism group and topological properties of graphs. In particular, the classes of finite vertex-transitive graphs embeddable on a given surface and those with bounded Hadwiger number have been asymptotically classified; the proof of the latter involves an excursion into hyperbolic plane geometry. Strong implications of bounds on the Hadwiger number (largest complete minor) to the structure of the automorphism group have been derived. In particular, a 30-year-old conjecture of the speaker on the composition factors of the automorphisms groups of finite graphs with bounded Hadwiger number has been confirmed.

Degree Ramsey and On-Line Degree Ramsey Numbers

Douglas B. West, University of Illinois at Urbana-Champaign

Abstract: Say that a graph H forces G (and write $H \rightarrow G$) if every 2-coloring of $E(H)$ has a monochromatic copy of G . The classical Ramsey number $R(G)$ is the minimum number of vertices in a graph that forces G . Burr, Erdős, and Lovász [1976] introduced a more general notion. For any monotone parameter ρ , let the ρ -Ramsey number $\hat{\rho}(G)$ be $\min\{\rho(H) : H \rightarrow G\}$.

In a game or “on-line” version of Ramsey coloring, each edge presented by Builder must be colored immediately by Painter. Let $\mathcal{H}_k = \{H : \rho(H) \leq k\}$. The on-line ρ -Ramsey number $\hat{\rho}(G)$ is the minimum k such that Builder can force a monochromatic G when required to keep the presented graph in \mathcal{H}_k .

Note that $\rho(G) \leq \hat{\rho}(G) \leq \hat{\rho}(G)$. In this talk, we review past results on $\hat{\rho}(G)$ and $\hat{\rho}(G)$ when ρ is the clique number, the chromatic number, and the number of edges. We also present new results when ρ is the maximum degree.

A greedy strategy for Painter shows that for all graphs,

$$\hat{\Delta}(G) \geq \Delta(G) - 1 + \max_{xy \in E(G)} \min\{d(x), d(y)\}.$$

Upper bounds follow by proving that it suffices for Builder to have a winning strategy against a “consistent” Painter whose choice of color depends only on the 2-colored component(s) containing the ends of the presented edge (this applies also to other on-line parameter Ramsey numbers). Sample results include $\hat{\Delta}(G) \leq 2\Delta(G) - 1$ when G is a tree and $\hat{\Delta}(C_n) = 4$ for all but finitely many n . Results for $\hat{\Delta}(G)$ are weaker. The results are joint work with Jane Butterfield, Tracy Grauman, Bill Kinnersley, Kevin Milans, and Chris Stocker.

Contributed Talks

$(n, n + k)$ -Graphs that maximize the Integrity for small k

Mustafa Atici, Western Kentucky University

Abstract: (n, m) -graphs are connected simple graph with n vertices and m edges. It is already known that cycle C with n vertices has largest integrity in (n, n) family of graphs. So we determine the graphs that have maximum integrity in families of $(n, n + k)$ for small k values.

The A_4 -structure of a graph

Michael D. Barrus, University of Illinois at Urbana-Champaign

Abstract: We define the A_4 -structure of a graph G to be the 4-uniform hypergraph on $V(G)$ whose edges are vertex subsets inducing alternating 4-cycles. We show how the A_4 -structure of a graph relates to its degree sequence, to vertex sets of its matchings, and to its canonical decomposition (as defined by Tyshkevich), and we characterize graphs having the same A_4 -structure as a split graph.

Counting as an efficient method for computing the distinguishing and distinguishing chromatic numbers of graphs

Christine Cheng, University of Wisconsin-Milwaukee

Abstract: A vertex k -coloring of a graph G is *distinguishing* if the only automorphism of G that preserves the colors is the identity map. The *distinguishing number* of G , $D(G)$, is the smallest integer k so that G has a distinguishing k -coloring.

The usual method for proving that $D(G) = k^*$ is to present a k^* -distinguishing coloring of G and then argue that fewer than k^* colors are not sufficient for destroying all the automorphisms of G . In this talk, we will sketch a more “roundabout” method for computing $D(G)$ – instead of solving for $D(G)$ directly, we will count the number of inequivalent distinguishing k -colorings of G , $D(G, k)$, and find the smallest k such that $D(G, k) > 0$. This is similar to the idea of counting the number of proper k -colorings of a graph to determine its chromatic number. What is interesting is that this technique leads to efficient algorithms for computing the distinguishing numbers of rooted trees, planar graphs and interval graphs. We shall also describe how it can be used to determine the distinguishing chromatic numbers of graphs.

A generating function approach to the surface area of some interconnection networks.

Eddie Cheng, Oakland University

Abstract: An important and interesting parameter of an interconnection network is the number of vertices of a specific distance from a specific vertex. This is known as the surface area or the Whitney number of the second kind. In this talk, we give explicit formulas for the surface area of the (n, k) -star graphs and the arrangement graphs via the generating function technique. As a direct consequence, it will also provide such explicit formulas for the star graphs, the alternating group graphs and the split-stars. Joint work with Ke Qiu and Zhizhang Shen.

A decomposition problem of regular hypergraphs

Jeong Ok Choi, Furman University

Abstract: An r -block is a 0,1-matrix in which every row has sum r . Let S_n be the set of pairs (k, l) such that the columns of any $(k + l)$ -block with n rows split into a k -block and an l -block. We determine S_n for $n \leq 5$. In particular, $S_3 = \{(k, l) : 2 \mid kl\}$, $S_4 = \{(k, l) : (6 \mid k \text{ or } l) \text{ and } (1 \notin \{k, l\})\}$, and $S_5 = \{(k, l) : 11 \neq \min\{k, l\} > 7 \text{ and each value in } \{3, 4, 5\} \text{ divides } k \text{ or } l\}$. The problem arose from a list-coloring problem in digraphs and is a refinement of the notion of indecomposable hypergraphs. This is joint work with Douglas B. West.

Is the independent set sequence of the hypercube unimodal?

David Galvin, University of Notre Dame

Abstract: Let a_n be the number of independent sets of size n in a graph. It is conjectured that in many cases (for example, when the graph is bipartite) the sequence $\{a_n\}$ is unimodal, but not a great deal is known. I’ll talk about work in progress to asymptotically enumerate the number of independent sets of fixed size in the discrete hypercube $\{0, 1\}^d$, leading to a partial unimodality result in this case.

Destroying cycles in digraphs

Peter Hamburger, Western Kentucky University

Abstract: For a simple directed graph G , let $\beta(G)$ be the size of the smallest subset $X \in E(G)$ so that $G \setminus X$ has no directed cycles, and let $\gamma(G)$ denote the number of unordered pairs of nonadjacent vertices in G . Chudnovsky, Seymour, and Sullivan showed that $\beta(G) \leq \gamma(G)$, and conjectured that $\beta(G) \leq \frac{\gamma(G)}{2}$. We prove that $\beta(G) < 0.88\gamma(G)$. This is joint work with Molly Dunkum and Attila Pór, Department of Mathematics and Computer Science, Western Kentucky University.

Spinor Components and Graph Theory

Glenn Harris, Southern Illinois University Edwardsville

Abstract: Spinors were first discovered by Elie Cartan in 1913. Since then spinors have been useful in Quantum Mechanics and studied extensively by mathematical physicists. Connections between mathematical physics and graph theory have only recently become topics of study. The approach of this project is to start with an arbitrary simple graph, associate it with a spinor, and use the properties of the spinor to reveal properties of the graph. In particular, the properties of the spinor reveal the number of k -matchings of the graph. My goal is to find as many connections between spinors and their associated graphs as possible. This project is a collaboration with Dr. Staples of SIUE.

The clique-separator graph of a chordal graph

Lou Ibarra, DePaul University

Abstract: We present a new representation of a chordal graph called the clique-separator graph, whose nodes are the maximal cliques and minimal vertex separators of the graph. We present structural properties of the clique-separator graph and additional properties when the chordal graph is an interval graph, proper interval graph, or split graph.

Compact topological cliques in sparse graphs

Tao Jiang, Miami University

Abstract: Let ϵ be any real number such that $0 < \epsilon < 1$. Answering a question of Paul Erdős, Kostochka and Pyber (1988) showed that for large n , every n -vertex graph with at least $4^{t^2} n^{1+\epsilon}$ edges contains a subdivision of K_t in which each edge of K_t is subdivided at most $c \log t/\epsilon$ times, where c is an absolute constant.

Here we prove a complementary (and in some sense stronger) result by eliminating the dependency on t . For each t and sufficiently large n , we show that every n -vertex graph with at least $a(t)n^{1+\epsilon}$ edges, where $a(t)$ is a constant depending on t , contains a subdivision of K_t in which each edge of K_t is subdivided at most $c \log(1/\epsilon)/\epsilon$ times, where c is an absolute constant. Note that the number of times each edge is subdivided depends only on ϵ and does not depend on t . This is a preliminary report.

A Survey of Iterated Distance Graphs

Garry Johns, Saginaw Valley State University

Abstract: Given a graph G , the antipodal graph $A(G)$ is formed by joining two vertices of G if the distance between them is the diameter of G . The sequence of iterated antipodal graphs includes the graph $G = A^0(G)$ followed by the graphs $A^n(G) = A(A^{n-1}(G))$ for $n \geq 1$. In this talk, properties of this sequence - including limits and periodicity - will be given and open questions will be shared. Similar results will also be discussed for eccentricity digraphs and detour antipodal graphs (two variations of antipodal graphs).

On the Independent Domination Number of a Random Graph

Darin Johnson, Southern Illinois University, Carbondale

Abstract: We prove a two point concentration for the independent domination number of the random graph $G_{n,p}$ provided $p^2 \ln(n) \geq 64 \ln((\ln n)/p)$. We also discuss the distribution of the number of independent dominating sets of size $s = 1$ to n in $G_{n,p}$ and give some empirical data to support a conjecture.

Choosability with separation in complete graphs

Mohit Kumbhat, University of Illinois at Urbana-Champaign

Abstract: Let $\phi(G, c)$ be the minimum value of k such that G is k -list colorable with the lists $L(v)$ satisfying $|L(u) \cap L(v)| \leq c$ for all $uv \in E(G)$. Kratochvíl et al. posed the question about finding $\lim_{n \rightarrow \infty} \phi(K_n, c)/\sqrt{cn}$. We prove that the limit is 1. This is joint work with Z. Füredi and A.V. Kostochka.

Finding a Biplanar Imbedding of $C_n \times C_n \times C_l \times P_m$

Josh Lambert, North Dakota State University

Abstract: Determining the biplanar crossing number of the graph $C_n \times C_n \times C_n \times P_n$ was a problem proposed in a paper by Czaparka, Sýkora, Székely, and Vrtó. We find as a corollary to the main theorem of this talk that the biplanar crossing number of the aforementioned graph is zero. This result follows from the decomposition of $C_n \times C_n \times C_l \times P_m$ into one copy of $C_{n^2} \times P_{lm}$, $l - 2$ copies of $C_{n^2} \times P_m$, and a copy of $C_{n^2} \times P_{2m}$.

Group Colorability of Multi-graphs

Hao Li, West Virginia University

Abstract: Let G be a multi-graph with a fixed orientation D and let Γ be a group. Let $F(G, \Gamma)$ denote the set of all functions $f : E(G) \rightarrow \Gamma$. Then G is Γ -colorable if and only if for every $f \in F(G, \Gamma)$ there exists a Γ -coloring $c : V(G) \rightarrow \Gamma$ such that for every $e = uv \in E(G)$ (assumed to be directed from u to v), $c(u)c(v)^{-1} \neq f(e)$. We define the group chromatic number $\chi_g(G)$ to be the minimum number m for which G is Γ -colorable for any group Γ of order $\geq m$ under the orientation D . In this paper, we investigated the properties of $\chi_g(G)$ and found an upper bound of group chromatic number for multi-graphs, especially for K_5 -minor free graphs and $K_{3,3}$ -minor free graphs.

On s -Hamiltonian-connected line graphs

Yanting Liang, West Virginia University

Abstract: A graph G is Hamiltonian-connected if any two of its vertices are connected by a Hamiltonian path (a path including every vertex of G); and G is s -Hamiltonian-connected if the deletion of any vertex subset with at most s vertices results in a Hamiltonian-connected graph. In this paper, we prove that the line graph of a $(t + 4)$ -edge-connected graph is $(t + 2)$ -Hamiltonian-connected if and only if it is $(t + 5)$ -connected, and for $s \geq 2$ every $(s + 5)$ -connected line graph is s -Hamiltonian-connected.

Complexity of Optimal Accumulation of Partial Derivatives on DAGs

Andrew Lyons, University of Chicago/Argonne National Laboratory

Abstract: I will discuss the evaluation of particular collections of expressions derived from edge-labeled directed acyclic graphs, with an emphasis on the degree to which the complexity of evaluating them optimally arises out of the structure of the graph. This problem has its origins in the field of automatic (or algorithmic) differentiation, where the graph G represents the computation of some vector function F and the aforementioned expressions result from algorithmic application of the chain rule to obtain some partial derivative information of F . I will give an overview of those cases where this problem is known to be NP-hard, as well as a brief description of a particular class of graphs that admits a polynomial time solution. If time permits, I will also present some results related to heuristics for this problem.

Unavoidable Minor Structures in Large Graphs

John Maharry, Ohio State University

Abstract: There have been a number of recent results roughly stating that sufficiently large graphs must contain certain graphs or families of graphs as minors under certain conditions. These conditions include high connectivity, high edge density, and high genus. This talk will be a survey of these results.

Graphs with complete minimal k -vertex separators

Terry McKee, Wright State

Abstract: G. A. Dirac characterized chordal graphs as those in which minimal vertex separators always induce complete subgraphs. I generalize that traditional (2-)vertex separator to a k -vertex separator—meaning a set of vertices whose removal disperses k independent vertices from one component into k separate components. Generalizing Dirac's theorem, the graphs in which minimal k -separators always induce complete subgraphs are the $\{P_5, 2P_3\}$ -free chordal graphs. I also discuss other features of k -vertex separators, including their appearance in clique trees of chordal graphs.

Cycle Spectra of Hamiltonian Graphs

Kevin Milans, University of Illinois at Urbana-Champaign

Abstract: The cycle spectrum of a graph G is the set of lengths of cycles in G . Let $s(G)$ denote the size of the cycle spectrum of G . Jacobson and Lehel asked for lower bounds on $s(G)$ when G is a 3-regular Hamiltonian graph on n vertices. They noted that $s(G)$ is at least $c \log(n)$ for some constant c and constructed a family of 3-regular Hamiltonian graphs on n vertices with $n/6 + 3$ distinct cycle lengths.

We improve Jacobson and Lehel's observation by show that if G is a Hamiltonian graph on n vertices with m edges, then $s(G)$ is at least $\sqrt{(m-n)/3}$. When n is even, the complete bipartite graph $K_{n/2, n/2}$ provides an example where $s(G)$ is at most $\sqrt{m-n} + 1$. Nevertheless, we conjecture that when G is Hamiltonian and 3-regular, $s(G)$ is at least cn for some constant c . This is joint work with Dieter Rautenbach, Friedrich Regen, and Douglas B. West.

Balloons, Cut-edges, Matchings, and Total Domination number in Regular Graphs of Odd Degree

Suil O, University of Illinois at Urbana-Champaign

Abstract: A *balloon* in a graph G is a maximal 2-edge-connected subgraph of G that is incident to exactly one cut-edge of G . By maximizing the number of balloons in a connected $(2r+1)$ -regular graph with n vertices, the maximum number of cut-edges and minimum size of a maximum matching in such graphs can be determined, and the graphs with the smallest maximum matchings can be characterized. Balloons also help in the study of the total domination number $\gamma_t(G)$. For a 3-regular graph G with n vertices and b balloons, we show that $\gamma_t(G) \leq (n/2) - (b/2)$ (with a slight exception when $b = 3$). This improves the known inequality $\gamma_t(G) \leq \alpha'(G)$ for 3-regular graphs, where $\alpha'(G)$ is the matching number, since the results on balloons yield $\alpha'(G) \geq (n/2) - (b/3)$. This is joint work with Douglas West.

Environmental Evolutionary Graph Theory

Gregory Puleo, Rochester Institute of Technology

Abstract: We consider a simple spatial model of competition between two species. The environment is represented by a graph with red and blue vertices, which offer different levels of reproductive fitness to the two species. In general, the process appears to be difficult to analyze. However, in the case where the coloring of the vertices is a "proper" two-coloring, we show that these graphs are fair: neither species has an overall advantage.

H-Matchable Trees: Degree and Distance Distribution

Andrew M. Schwartz, Southeast Missouri State University

Abstract: Fix a nontrivial tree H . We call $\{H_1, \dots, H_n\}$ a perfect H -matching of a tree T if and only if H_1, \dots, H_n are subtrees of T where H_i is isomorphic to H and $\{V(H_1), \dots, V(H_n)\}$ partitions $V(T)$. Then a perfect P_2 -matching is a perfect matching of T . A tree is H -matchable if and only if it has a perfect H -matching. We present some formulas for the number of labeled H -matchable trees, the degree distribution of a random vertex in a random labeled H -matchable tree, and the distance distribution of two random vertices in a random labeled matchable tree. We also prove some asymptotic results of these formulas.

Connectivity and Minimum Degree of Iterated Line Graphs

Yehong Shao, Ohio University Southern

Abstract: Let $k \geq 0$ be an integer and $L^k(G)$ be k -th iterated line graph. Niepel, Knor, and Šolteš conjectured that for any connected graph G that is not a path, there exists an integer K such that, for all $i \geq K$, $\delta(L^{i+1}(G)) = 2\delta(L^i(G)) - 2$. Niepel, Knor, and Šolteš also proved if this conjecture is true, then for any connected graph G that is not a path, there exists an integer K such that, for all $i \geq K$, $\kappa(L^{i+1}(G)) = 2\kappa(L^i(G)) - 2$. Hartke and Higgins in 2003 proved the conjecture of Niepel, Knor, and Šolteš using induced subgraphs of minimum degree vertices and locally minimum vertices. We give a different proof of $\kappa(L^{i+1}(G)) = 2\kappa(L^i(G)) - 2$ using Hartke and Higgins's result.

Connectivity properties and orientation of generalized Cayley graphs generated by transpositions

Nart Shawash, University of Detroit

Abstract: Star graphs S_n^* are Cayley graphs on a symmetric group S_n with the generator set consisting of transpositions $(1, 2), (1, 3), \dots, (1, n)$. In the eighties, S_n^* were proposed as an attractive alternative to hypercubes. The major disadvantage for interconnection networks using S_n^* topology is the number of vertices $n!$. As a remedy, two closely related graphs were proposed in the literature, arrangement graphs $A_{n,k}$ and (n, k) -star graphs $S_{n,k}$, both with $\frac{n!}{(n-k)!}$ vertices.

This talk introduces the graph $Q_{n,k,m}$ on $\frac{n!}{(n-k)!}$ vertices, which is a generalization of a Cayley graph on S_n generated by an arbitrary transposition set in a similar fashion to $A_{n,k}$ and $S_{n,k}$ generalization of S_n^* .

The notion of *tightly-super-connectedness* was introduced in 1981 by D. Bauer et al in the context of connectivity extremal problems. We show that $Q_{n,k,m}$ is tightly-super-connected.

Digraphs are important in the design of interconnection networks with unidirectional data flow. Local orientation rules of $Q_{n,k,m}$ resulting in a small diameter is derived. (This is joint work with Eddie Cheng of Oakland University.)

A generalization of the Friendship Theorem

Jian Shen, Texas State

Abstract: The Friendship Theorem states that if any two people in a party have exactly one common friend, then there exists a politician who is a friend of everybody. In this talk, we generalize the Friendship Theorem. Let λ be any nonnegative integer and μ be any positive integer. Suppose each pair of friends have exactly λ common friends and each pair of strangers have exactly μ common friends in a party. The corresponding graph is a generalization of strongly regular graphs obtained by relaxing the regularity property on vertex degrees. We prove that either everyone has exactly the same number of friends or there exists a politician who is a friend of everybody. As an immediate consequence, this implies a recent conjecture by Limaye et al.

A criterion for embeddability in the projective plane

Despina Stasi, University of Illinois at Chicago

Abstract: We show that a graph can be embedded in the projective plane if it can be drawn with all pairs of non-adjacent edges crossing evenly. This result extends the Hanani-Tutte theorem from the plane to the projective plane.

An unavoidable set of D-reducible configurations

John Steinberger, University of British Columbia

Abstract: We give a new proof of the four color theorem by exhibiting an unavoidable set of 2832 D-reducible configurations. This settles a 1975 conjecture of Stromquist reiterated by Roberston, Sanders, Seymour and Thomas.

A bound on the domination number of connected cubic graphs

Christopher Stocker, University of Illinois at Urbana-Champaign

Abstract: Let G be an n -vertex 3-regular graph. Kostochka and Stodolsky proved that the domination number of G is at most $4n/11$ when $n \geq 11$. We describe the techniques used to further improve this bound to $5n/14$, which is sharp for $n = 14$. This is joint work with Alexandr Kostochka.

The boolean number of a graph

Bridget Tenner, DePaul University

Abstract: In this talk, we will examine the homotopy type of the boolean complex of a graph. In particular, we will illustrate connections between these objects and several well known sequences including the number of derangements, the Genocchi numbers of the second kind, and the Legendre-Stirling numbers.

Connected Decycling

Robert (Chip) Vandell, Indiana Purdue University Fort Wayne

Abstract: A decycling set of a graph G is a subset S of the vertex set, such that $G - S$ is acyclic. The decycling number of a graph is the minimum order of a decycling set. In this talk we will investigate what happens to the decycling number if we add the constraint that the decycling set must be connected.

The Circle Space: Zero-Weighted Cycles in a Weighted, Undirected Graph

William Vautaw, Saginaw Valley State University

Abstract: Given a connected, undirected graph G , we consider the vector space $C(G) = \{w : E \rightarrow R \mid w(H) = 0 \text{ for all cycles } H \text{ in } G\}$. We call $C(G)$ the circle space of G . By defining an equivalence relation on the edge set E of G , we determine precisely which weightings w lie within $C(G)$, and, in so doing, the dimension of $C(G)$. We present an application of this idea in the topological theory of surface mapping class groups.

Finite Euclidean and non-Euclidean graphs

Le Anh Vinh, Harvard University

Abstract: A classical set of problems in combinatorial geometry deals with the questions of whether a sufficiently large subset of R^d , Z^d or F^d contains a given geometric configuration. Examples are Erdős distance problem, Szemerédi-Trotter theorem and the Furtenberg-Katznelson-Weill theorem. In the finite non-Euclidean spaces, however, the use of known methods, like incidence geometry or Fourier analysis, is nontrivial to the author. We therefore approach the problems using graph theoretic method. Our main tools are graphs associated to the finite Euclidean and non-Euclidean spaces. The advantages of using these graphs are twofold. First, we can reprove and sometimes improve several known results. Secondly, our approach works transparently in the non-Euclidean setting. Due to time constraints, I will only restrict our discussion to the Erdős distance problems and a Furtenberg-Katznelson-Weill type theorem over finite Euclidean and non-Euclidean spaces.

The Paranoid Watchman: a search problem on graphs

Matt Walsh, Indiana University-Purdue University Fort Wayne

Abstract: A watchman is touring a graph to ensure that it is free of intruders. He is aware of any intruders within his immediate neighbourhood (i.e. within distance 1 of his position) and wishes to find a route that will guarantee that any intruders will be detected. We give necessary and sufficient conditions for such a route to exist in a given graph, and examine some generalizations of the problem. We also compare this with other search problems on graphs, such as cops & robbers and domination search. This is joint work with C. D. Leach (University of West Georgia) and J. Dalzell (Ivy Tech Community College).

Fixing sets and resolving sets for the rook's graph

W. Doug Weakley, Indiana University - Purdue University at Fort Wayne

Abstract: Let G be a connected simple graph.

A *fixing set* (also called a *determining set*) of G is a set S of vertices of G such that the only automorphism of G that fixes all vertices in S is the identity. The minimum size of a fixing set of G is the *fixing number* of G , denoted $\text{fix}(G)$.

A *resolving set* (also called a *reference set* or *locating set* or *distance determining set*) of G is a set $\{v_1, \dots, v_k\}$ of vertices of G such that for any vertex v of G , the k -tuple $(d(v_1, v), \dots, d(v_k, v))$ of distances determines v . The minimum size of a resolving set of G is the (*metric*) *dimension* of G , denoted $\text{dim}(G)$.

Every resolving set is a fixing set, so $\text{fix}(G) \leq \text{dim}(G)$. D. Boutin has asked whether $\text{dim}(G) - \text{fix}(G)$ can be arbitrarily large.

Let R_n denote the *rook's graph*; its vertices are the squares of the $n \times n$ chessboard, with two squares being adjacent if they share a row or column. (Therefore $R_n \cong K_n \square K_n$.) We show $\text{fix}(R_n) = n$ for $n > 1$ and $\text{dim}(R_n) = (4n - j - 3)/3$ where j in $\{-1, 0, 1\}$ satisfies $n \equiv j \pmod{3}$, thus answering Boutin's question affirmatively.

The Partial Acquisition Number of Graphs

Paul Wenger, University of Illinois at Urbana-Champaign

Abstract: Begin with a graph G with weight 1 on each vertex. For a vertex u with a neighbor v whose weight is at least the weight on u , an *acquisition move* transfers all of the weight from u to v . The *acquisition number* of G , written $a(G)$, is the minimum number of vertices with positive weight after a sequence of acquisition moves in G .

In this talk we introduce the following generalization of acquisition on graphs. For a vertex u with a neighbor v whose weight is at least the weight on u , a *partial acquisition move* transfers an integer amount of the weight on u from u to v . The *partial acquisition number* of G , denoted $a_p(G)$, is the minimum number of vertices with positive weight after a sequence of partial acquisition moves in G . We present results on the partial acquisition number of trees and graphs with diameter two. This is joint work with Noah Prince.

Bounded number of components of 2-factors in line graphs

Huiya Yan, West Virginia University

Abstract: Let G be a graph of order n . A 2-factor is a 2-regular spanning subgraph of a graph G . A lot of results on the components of a 2-factor in G have appeared by studying the conditions on the minimum degree of the graph G . In this paper we avoid studying the minimum degree and get the following: if $\max\{d(x), d(y)\} \geq (n - \mu)/p - 1$ holds for any $xy \notin E(G)$ and $|U| \neq 2$, where $U = \{v : d(v) < (n - \mu)/p - 1\}$, p is a positive integer and μ is a nonnegative integer, then for n sufficiently large relative to p and μ , $L(G)$ has a 2-factor with at most $p + 1$ components. Moreover, $L(G)$ has a 2-factor with at most p components if $|U| \leq 1$. Especially, it extends a result of Z. Niu and L. Xiong saying that if $\delta(G) \geq n/p - 1$, i.e., $U = \emptyset$, then $L(G)$ has a 2-factor with at most p components. We also show the graphs above are $(p + 2)$ -supereulerian, i.e., have a spanning even subgraph with at most $p + 2$ components. All results are best possible.