# Math 453 – Combinatorics

**Course Description from Bulletin:** Permutations and combinatorics; pigeonhole principle; inclusion-exclusion principle; recurrence equations and generating functions; enumeration under group action. (3-0-3)

**Enrollment:** Elective for AM and other majors.

**Textbook(s):** Fred S. Roberts and Barry Tesman,*Applied Combinatorics*, $2^{nd}$ Edition, Pearson Prentice Hall (2005).

**Other required material:** None

**Prerequisites:** MATH 230 Introduction to Discrete Mathematics, or consent of the instructor

**Objectives:**

1. Students will be able to generate conjectures from examples and formulate precise conjectures.
2. Students will be able to recognize and write valid proofs. Proof techniques include bijective/combinatorial proofs, induction, and the pigeonhole principle.
3. Students will be able to discuss mathematics, including: presenting solutions at the board, generating examples for illustration as appropriate, seeking and finding holes in proposed proofs.
4. Students will be familiar with common examples including: subsets of a set, functions, onto functions, one-to-one functions, basic graph substructures, Lattice Paths and Catalan Numbers, the Binomial Theorem and Pascal's Triangle, basic Ramsey Numbers, Fibonacci Numbers, multisets and compositions of integers, derangements, colorings of n-gons up to symmetry.
5. Students will be able to count (with proofs) standard and unfamiliar examples using the techniques listed below. Also, be familiar with (proof of) why each technique is valid.
    a. sum rule, product rule, quotient rule
    b. distributions from "the 20-fold way" including permutations, combinations, multisets, compositions of integers, Stirling numbers of the second kind, and partitions of integers
    c. recurrence relations and ordinary generating functions
    d. Principle of Inclusion-Exclusion
    e. groups acting on sets ("Burnside's Lemma" and "Polya Enumeration")

**Lectures:** 3 50 minute classes per week, or 2 75 minute classes per week.

| **Course Outline:** | Hours |
|---|---|
| 1. Basic Counting Principles and Examples | 6-9 |
| 2. Applications of Induction and Recursion in Combinatorics | 4-10 |
| 3. Distribution Problems | 6-9 |
| 4. Generating Functions | 6-9 |
| 5. The Principle of Inclusion-Exclusion | 3-5 |

6. Groups Acting on Sets 4-10

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| **Assessment**: | Problem Solutions | 10% |
| | Quizzes | 10% |
| | Midterm Exams | 50% |
| | Final Exam | 30% |

**Alternatively, make the following changes:**

**Textbook(s):** Kenneth P. Bogart, *Enumerative Combinatorics Through Guided Discovery*, manuscript, March 20, 2005.

**Lectures:** None. The class will be taught as a "Guided Discovery" course. This means that the primary text for the course is a set of problems with just enough prose so that the problems will make sense. By working through these problems, students will discover for themselves the main theorems of combinatorial mathematics (and their proofs) and examples of how these theorems are used. This method has been tested at several universities across the country, and the result has been a dramatic increase in student understanding, a result supported by current research on how humans learn mathematics. In addition, the approach develops *problem-solving* ability, a highly-prized skill which includes: reading and interpreting what is being asked, the ability to verbalize questions, the willingness to consider different ideas and theories, the courage to experiment with those ideas, the ability to endure frustration and failure, the wisdom and experience to understand when an approach is not succeeding, and the ability to recognize a solution.

**Class Format:** Classes will be devoted to a discussion of the problems and the broad themes in combinatorial mathematics the problems illustrate. This will include small group discussions, student presentations of solutions and ideas, and whole class discussion. The groups will not necessarily stay constant throughout the semester, and the instructor may change them around sometimes. Students must actively participate in class.

The instructor will occasionally give a short presentation: a recap before starting a new topic, to place results in context, or give a different perspective.

**The Problems:** Roughly 15-20 problems are assigned each week to work through. Except when otherwise specified, complete solutions to all of these problems must be written up. Students are encouraged to work together on problems, both in class and outside of class. Students may discuss solutions and develop outlines for writing up solutions. However, when students write the final draft of a problem they **should not** work together with other students, and they should turn in their final draft without showing it to anyone else.

Certain problems will be designated as "mandatory" problems. These problems will be graded by the instructor and returned to the student for revision as necessary. A student is expected to work on these problems until they are essentially correct.

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| **Assessment**: | Problem Solutions | 50% |

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| Active Class Participation | 15% |
| Midterm Exam | 10% |
| Final Exam | 25% |

"Problem Solutions"

Problems that are correct and well-written will be awarded ten points. Problems that are almost there will be awarded nine points, and students need not continue working on that problem to get ten points if he or she chooses not to. Problems on which a student has made significant progress will be awarded five points, but students are expected to continue working on such problems. It is worthwhile to find more than one solution to a problem, and so students will receive bonus points for additional (distinctly different) solutions to a problem.

Careful grading of the problems is a very time-consuming process. Students are asked to facilitate the process by keeping their problems in a loose-leaf notebook with a table of contents (an appropriate form will be provided) that shows when problems were submitted, what page they are on, when revisions were submitted and what page they are on. Revisions should not be made by writing on the original problem, since part of the grading process is comparing the original and revised versions of the problems. Problem notebooks will be collected once a week. (If there's a problem on which you would like to continue working, you may wish to photocopy it before turning in your notebook.)

The book has appendices on mathematical induction, functions, and relations. Students who need to work through these appendices will not need to submit as many problems from the regular chapters. Students who wish to skip to more challenging problems may substitute them for other problems. In both cases, how exactly to do this must be agreed upon with the instructor beforehand.

It is not expected to complete all of the problems in the book. However, anyone completing 90% of the problems in the book (and not by avoiding the most challenging 10%) will get the full credit for "Problem Solutions".

"Active Class participation"

Students will be asked to come to class ready to present their homework solutions, and sometimes ideas or partial solutions. Students must volunteer to present problems regularly. Listeners will ask for clarifications as needed, and all must be ready to participate in ensuing discussions. Students must also contribute to their small group discussions. Since coming late to class may disrupt small group discussions, this will be reflected in the class participation grades of students who habitually come late to class.

**Syllabus prepared by**: Michael Pelsmajer and Robert Ellis
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