

Friday, March 13, 2015

On sheet of paper covered with an uniform grid of side-length 1 we arbitrarily draw a circle of radius 10. Prove that at least 250 grid-nodes are inside of this circle.

Solution. Around each node of the grid plot a unit square with edges parallel to the original grid. Consider the collection S of squares whose centers are inside the circle; the number of these squares, say X , is the same as the number of grid-nodes inside the circle.

Next consider a circle concentric with the original circle of radius 9. The squares in S will cover the entire circular disk of radius 9 (since the distance from the center of a square to one of its corners is $\frac{\sqrt{2}}{2} < 1$). Thus the total area of all the squares in S which is equal to X is larger than the area of the circle of radius 9:

$$X \geq 81\pi > 251.$$

Good Luck! Have fun and enjoy Mathematics!