

Friday, March 06, 2015

Sum the infinite series $\sum_{i=1}^{\infty} \frac{1}{(3i-2)(3i+1)}$

Solution. By the method of partial fraction decomposition we find that

$$\frac{1}{(3i-2)(3i+1)} = \frac{1/3}{3i-2} - \frac{1/3}{3i+1}.$$

Hence

$$\begin{aligned} \sum_{i=1}^n \frac{1}{(3i-2)(3i+1)} &= \frac{1}{3} \sum_{i=1}^n \left(\frac{1}{3i-2} - \frac{1}{3i+1} \right) \\ &= \frac{1}{3} \left(\frac{1}{1} - \frac{1}{3n+1} \right) \end{aligned}$$

because the summation telescopes. Thus,

$$\sum_{i=1}^{\infty} \frac{1}{(3i-2)(3i+1)} = \lim_{n \rightarrow \infty} \frac{1}{3} \left(\frac{1}{1} - \frac{1}{3n+1} \right) = \frac{1}{3}$$

Good Luck! Have fun and enjoy Mathematics!