Friday, February 27, 2015

Let

$$a_n = \frac{1^{\frac{7}{2}} + 2^{\frac{7}{2}} + \cdots + n^{\frac{7}{2}}}{n^{\frac{9}{2}}} .$$

Show that $\lim_{n\to\infty} a_n$ exists and find its value.

Solution. The limit is $\frac{2}{9}$. Write a_n in the form

$$a_n = \frac{1}{n}\left[\left(\frac{1}{n}\right)^{\frac{7}{2}} + \left(\frac{2}{n}\right)^{\frac{7}{2}} + \cdots \left(\frac{n}{n}\right)^{\frac{7}{2}}\right],$$

and recognize this as a Riemann sum for the integral

$$\int_0^1 x^{\frac{7}{2}}\, dx = \frac{2}{9}x^{\frac{9}{2}}\Big|_0^1 = \frac{2}{9}.$$

Good Luck! Have fun and enjoy Mathematics!