

**Friday, February 20, 2015**

A semi-trailer has two tires on the front wheels (one tire on each side), and four tires on the rear wheels (two on each side). The front tires are worn in 15,000 miles, while the back ones are worn in 25,000 miles. How should the driver swap the tires to get the maximum driving distance? Find that distance.

**Solution.** The maximum distance is equal to 20,454.54 miles. The driver has to swap the tires such that each tire is on the front wheels  $1/3$  of this distance.

Let us consider as one unit the amount of rubber needed to be lost before the tire gets worn. Thus, at the beginning we have 6 units of rubber, and during one mile of driving  $1/15000$  units are worn if the tire is in the front, and  $1/25000$  if it is in the back. Hence, during one mile drive  $2/15000 + 4/25000$  units of rubber are lost. Clearly that after  $x$  miles we have lost

$$\frac{11x}{37500}$$

units of rubber, and this amount can not be bigger than 6. Solving for  $x$ , we get

$$x \leq \frac{6 \times 37500}{11} = 20454.54545454545.$$

Finally, the driver needs to divide the driven amount equally among the set of 6 tires, i.e. each tire is  $1/3$  of the road on the front wheels.

Good Luck! Have fun and enjoy Mathematics!