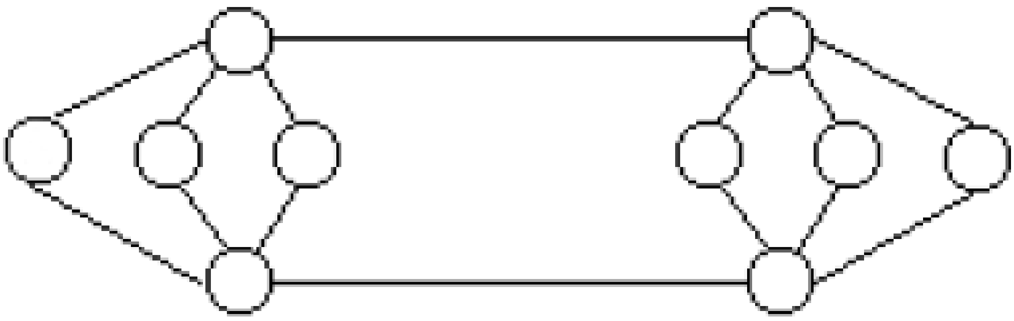
Illinois Institute of Technology
Department of Applied Mathematics and IIT SIAM Student Chapter

Weekly Problem Competition

Friday, September 11, 2009

Choose 10 different numbers from $\{0, 1, 2, \cdots, 14\}$ and put them into the following circles. If there is an edge between two circles, then take the absolute value of their difference. Is it possible to have 14 different absolute values?



Remarks:
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You have to submit the solution by email, to `weeklyproblem@math.iit.edu`
Please feel free to tell any IIT undergraduate student about the competition.

Thank you for your participation
Good Luck !

Illinois Institute of Technology
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Weekly Problem Competition

Friday, September 18, 2009

Let $a_1, a_2, \ldots, a_n$ be n positive numbers, such that their product is equal to 1. Show that

$$(1 + a_1)(1 + a_2) \ldots (1 + a_n) \geq 2^n \ .$$

Remarks:
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Good Luck !

Illinois Institute of Technology
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Weekly Problem Competition

Friday, September 25, 2009

Suppose that n is natural number and $2n^2$ is divisible by d. Prove that $n^2 + d$ is not a perfect square.

Remarks:
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Good Luck !

Illinois Institute of Technology
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Weekly Problem Competition

Friday, October 2, 2009

A set of alphabetical blocks has a single different letter of the alphabet on each of the six sides of each block. In all, the four blocks contain 24 letters of the alphabet. By arranging the blocks in various ways, you can spell all the following words. Can you figure out how the letters are arranged on the four blocks?

$$BOWL, LYNX, DEAL$$
$$MICA, FUSE, NECK$$
$$GORE, RUNT, INCH$$
$$WHEY, JUMP, ZIPS$$

Remarks:
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Weekly Problem Competition

Friday, October 9, 2009

Find the limit

$$\lim_{n\to\infty}\left[(1+\frac{1}{n^2})(1+\frac{4}{n^2})(1+\frac{9}{n^2})\cdots(1+\frac{n^2}{n^2})\right]^{\frac{1}{n}}.$$

Remarks:
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Good Luck !

Illinois Institute of Technology
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Weekly Problem Competition

Friday, October 16, 2009

Assume that $a_1, a_2, \ldots, a_n$ are positive integers ($n \geq 2$), such that $a_1 < a_2 < \ldots < a_n$ and $\sum_{k=1}^{n} \frac{1}{a_k} \leq 1$. Prove that for any real number x, the following inequality holds true

$$\left(\sum_{k=1}^{n} \frac{1}{a_k^2 + x^2} \right)^2 \leq \frac{1}{2} \frac{1}{a_1(a_1 - 1) + x^2}.$$

Remarks:
The rules and results of the competition can be found at `http://www.math.iit.edu/~weeklyproblem`
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Weekly Problem Competition

Friday, October 23, 2009

Four 1's and five 0's are written on a circle in no particular order. We perform the following operation with these numbers: between same numbers we write a 0, and between different numbers we write an 1; then all original numbers are erased. With obtained numbers we perform the same operation again. Prove that after several such operations it is impossible to get nine zeros.

Remarks:
The rules and results of the competition can be found at `http://www.math.iit.edu/~weeklyproblem`
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Thank you for your participation
Good Luck !

Illinois Institute of Technology
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Weekly Problem Competition

Friday, October 30, 2009

Solve the equation

$$x! + y! + z! = u!$$

in positive integers, where $n!$ denotes the product of first n positive integers ($n! := 1 \cdot 2 \ldots n$).

Remarks:
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Thank you for your participation

Illinois Institute of Technology
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Weekly Problem Competition

Friday, November 6, 2009

Let $a_1, a_2, \ldots, a_n$ be positive real numbers that satisfy the following inequality

$$(a_1^2 + a_2^2 + \cdots + a_n^2)^2 > (n-1)(a_1^4 + a_2^4 + \cdots + a_n^4)$$

for some $n \geq 3$. Prove that any three of the numbers a_i's are edges of some triangle.

Remarks:
The rules and results of the competition can be found at `http://www.math.iit.edu/~weeklyproblem`
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Thank you for your participation
Good Luck !

Illinois Institute of Technology
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Weekly Problem Competition

Friday, November 13, 2009

Solve the following equation

$$\cos 24x = 5\sin 3x + 9\tan^2 x + \cot^2 x.$$

Remarks:
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Good Luck !

Illinois Institute of Technology
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Weekly Problem Competition

Friday, November 20, 2009

Assume that $x_0, \ldots, x_n$ are some real non-negative numbers such that $x_0 = 0$, and $\sum_{i=1}^{n} x_i = 1$. Prove that

$$1 \le \sum_{i=1}^{n} \frac{x_i}{\sqrt{1 + x_0 + x_1 + \cdots + x_{i-1}}\sqrt{x_i + \cdots + x_n}} < \frac{\pi}{2}.$$

Remarks:
The rules and results of the competition can be found at `http://www.math.iit.edu/~weeklyproblem`
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Weekly Problem Competition

Friday, January 29, 2010

You have a wooden ball and a piece of paper (large enough). You are allowed to use a compass to draw on the ball and you can use a compass and a ruler to draw on the paper. Plot on the piece of paper a circle of radius equal to the radius of the ball.

Remarks:
The rules and results of the competition can be found at `http://www.math.iit.edu/~weeklyproblem`
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Thank you for your participation
Good Luck !

Illinois Institute of Technology
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Weekly Problem Competition

Friday, January 29, 2010

The four vertices of rectangle P_1, P_2, P_3 and P_4 lie on the edges of triangle ABC. Prove that among the four triangles $\Delta P_1P_2P_3$, $\Delta P_1P_2P_4$, $\Delta P_1P_3P_4$, $\Delta P_2P_3P_4$, at least one of them has a area less than 1/4 of the ΔABC's.

Remarks:

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Good Luck !

Illinois Institute of Technology
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Weekly Problem Competition

Friday, February 12, 2010

Prove that there exists a function $f : \mathbb{N} \to \mathbb{N}$ such that

$$f(f(n)) = n^2, \quad n \in \mathbb{N}.$$

Remarks:
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Thank you for your participation
Good Luck !

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Weekly Problem Competition

Friday, February 13, 2009

Four people A, B, C, D are walking in the dessert. They have two 16oz bottles full of water, and only one 6oz cup (empty). How can they share the water so that everyone gets 8oz of water?

Remarks:
The rules and results of the competition can be found at `http://www.math.iit.edu/~weeklyproblem`
You have to submit the solution by email, to `weeklyproblem@math.iit.edu`
Please feel free to tell to any undergraduate student about the competition and thank you for your collaboration.

Thank you for your participation
Good Luck !

Illinois Institute of Technology
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Weekly Problem Competition

Friday, February 19, 2010

You are given 81 weights with corresponding masses $1^2, 2^2, \ldots, 81^2$. Divide these weights in three groups of equal mass, i.e. each group has the same combined weight.

Remarks:
The rules and results of the competition can be found at `http://www.math.iit.edu/~weeklyproblem`
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Thank you for your participation
Good Luck !

Illinois Institute of Technology
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Weekly Problem Competition

Friday, February 26, 2010

Prove that inequality

$$|a_0 + a_1 \cos(x) + a_2 \cos(2x) + \ldots + a_{2n+1} \cos((2n+1)x)| \geq |a_1 + a_2 + \ldots + a_{2n+1}|$$

has a real solution in x for any given reals $a_0, a_1, a_2, \ldots, a_{2n+1}$.

Remarks:
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Thank you for your participation
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Weekly Problem Competition

Friday, March 19, 2010

Find all functions $f : \mathbb{Z}^+ \to \mathbb{R}$ that satisfy the following identity

$$f(n+m) + f(n-m) = f(3n), \quad n, m \in \mathbb{Z}^+,\ n \geq m.$$

note: $\mathbb{Z}^+ := \{0, 1, 2, 3, 4, \ldots\}$

Remarks:
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Thank you for your participation
Good Luck !

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Weekly Problem Competition

Friday, March 26, 2010

Is it possible to place on the real line three intervals of even length such that the intersection of any two of them is of odd length?

Remarks:
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Illinois Institute of Technology
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Weekly Problem Competition

Friday, April 2, 2010

Assume that α and β are real numbers that satisfy the following relations

$$\cos(\alpha) = \beta, \qquad \cos(\beta) = \alpha \ .$$

Prove that $\alpha = \beta$.

Remarks:
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Weekly Problem Competition

Friday, April 9, 2010

The following numbers are written on the blackboard

$$1 \quad \frac{1}{2} \quad \frac{1}{3} \quad \frac{1}{4} \quad \frac{1}{5} \quad \frac{1}{6} \quad \frac{1}{7} \quad \frac{1}{8}.$$

Before each of these numbers you put arbitrarily a plus or a minus sign. Prove that the obtained algebraic sum is different from zero.

Remarks:
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