Illinois Institute of Technology

Department of Applied Mathematics and IIT SIAM Student Chapter

Weekly Problem Competition

Friday, September 12, 2008

Let X denote the set of first 37 natural numbers, $X = \{1, 2, \ldots, 37\}$. Is it possible to arrange the elements of X in a specific order such that

1. Every number is used once and only once;
2. The second number divides the first, the third number divides the sum of first two, and so on. The 37-th number in the arrangement divides the sum of first 36 numbers in the arrangement;
3. the first number is 37.

Is the arrangement unique?

Remarks:

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Good Luck !

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Weekly Problem Competition

Friday, September 19, 2008

10 students want to fulfill their empty cups with water from fountain(s). Suppose that the k-th student needs T_k minutes to fulfill his/her cup, and assume that $T_k \neq T_j$, for $k \neq j$.

1. If there is only one fountain, what should be the order of the waiting line such that the total time to fulfill all students cups is minimum? Prove your statement. Total time is the sum of waiting times in the line of all students, i.e. the time in the line spent by the 1st student, plus the time in the line spent by the second student, and so on, plus the time in the line spent by the last student.

2. Suppose that there are two water fountains. How should the students form the line to minimize the total waiting time. (i.e. Part 1 of the problem but two water fountains)

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Weekly Problem Competition

Friday, September 26, 2008

Jenna Hawk takes 5 three-credit hour courses in the fall semester and another 5 three-credit hour courses in the spring semester. In each semester she takes at least one math course and at least one non-math course. Her fall semester grade point average (GPA) for her math courses is above 3.5, but her spring semester GPA for her math courses is exactly 3.5. In the spring semester she gets a C in at least one of her non-math courses and her GPA for non-math courses in the spring is also lower than her GPA for non-math courses in the fall. Surprisingly, her overall GPA in the spring is greater than that in the fall. Explain how this is possible. How high can her spring non-math GPA possibly be? We assume that the grade system is A, B, C, D, E with corresponding point-values $4, 3, 2, 1, 0$.

Remarks:

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Weekly Problem Competition

Friday, September 26, 2008

On the graduation party, every boy must dance with at least one girl and every girl must dance with at least one boy. But there is no boy who danced with all girls and there is no girl who danced with all boys. Prove or disprove that among all the attendants, there are two boys and two girls such that each of these two boys danced only with one of these two girls and each of these two girls danced only with one of these two boys?

Remarks:

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Good Luck !

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Weekly Problem Competition

Friday, October 10, 2008

Find all real solutions of the following equation

$$4x^2 - 40[x] + 51 = 0\,,$$

where $[x]$ denotes the greatest integer less than x.

Remarks:

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Good Luck !

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Weekly Problem Competition

Friday, October 24, 2008

A factory got an order for 440 tubes of length a, 480 tubes of length b. There are 3 materials available, the first one can be cut into 4 tubes of length a, 8 tubes of length b and the cost is 60 dollars; the second one can be cut into 6 tubes of length a, 2 tubes of length b and the cost is 50 dollars; the third one can be cut into 4 tubes of length a, 4 tubes of length b and the cost is 40 dollars. How to purchase the 3 materials in order to minimize the cost?

Remarks:

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Good Luck !

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Weekly Problem Competition

Friday, October 31, 2008

Let x, y, z be non-negative real numbers satisfying $x + y + z = 1$. Show that

$$x^2y + y^2z + z^2x \leq \frac{4}{27},$$

and find when the equality holds true.

Remarks:

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Good Luck !

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Weekly Problem Competition

Friday, November 7, 2008

Find all integers n, $n > 1$, such that for any real numbers $x_1, x_2, ..., x_n$ whenever $\sum_{i=1}^{n} x_i = 0$, we have $\sum_{i=1}^{n} x_i x_{i+1} \leq 0$, where $x_{n+1} := x_1$.

Remarks:

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Good Luck !

Illinois Institute of Technology

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Weekly Problem Competition

Friday, November 21, 2008

This is the last weekly problem for this semester. We will resume the contest next semester.

Thanks for your participation.

Find all real values of the parameter $a \in \mathbb{R}$, such that

$$|2x - a| + |3x - 2a| \geq a^2$$

for all $x \in \mathbb{R}$.

Remarks:

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Good Luck !

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Weekly Problem Competition

Friday, January 30, 2009

Assume there are three houses A,B,C and the task is to find a point O equally distant from these houses. Suppose that that there exists such a point O, and you can measure the distance from O to each house, however you can not measure directly the distance between the houses (see schematic map from Figure 1).

The problem: Find a practical method of determining the point O.

While there are many methods, the solution will be judged by how easy is to implement the method in practice, how reliable (exact) will be the result, and as usually the first "correct" answer gets the prize. Describe each step of the developed method(s).

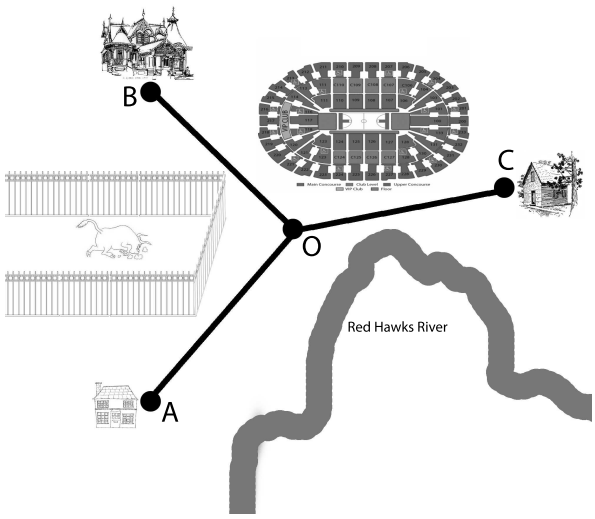

Figure 1: Schematic representation

Remarks:

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Thank you for your participation
Good Luck !

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Weekly Problem Competition

Friday, February 6, 2009

While looking at some books that you bought on the flee-market, you found a map of a treasure island. It states clear the location of the island and the instructions how to get to the treasure. The instructions read:

> From the chief's house walk toward the coconut tree, counting the number of steps. Turn 90° to the right and walk the same number of steps and mark the position. From the same house walk to the orange tree, count the steps, turn 90° to the left, walk the same distance, and mark the position. The treasure is located halfway between two marked points.

You locate the island using google-maps. Indeed, there is such an island. Moreover, google-maps provides the satellite view of the island. Good news - you found the coconut tree and the orange tree on the map. Bad news (or maybe not so bad) - there are no houses on the island.

The problem: find a method to locate the treasure using the above information. To dig the entire island is not feasible (the island is too big).

Hint: one solution assumes knowledge only in elementary geometry.

Remarks:
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Thank you for your participation
Good Luck !

Illinois Institute of Technology
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Weekly Problem Competition

Friday, February 13, 2009

Four people A, B, C, D are walking in the dessert. They have two 16oz bottles full of water, and only one 6oz cup (empty). How can they share the water so that everyone gets 8oz of water?

Remarks:
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Thank you for your participation
Good Luck !

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Weekly Problem Competition

Friday, February 20, 2009

Suppose that the sequence of real numbers $\{x_n\}_{n\in\mathbb{N}}$ is such that

$$|x_{n+m} - x_n - x_m| < \frac{1}{m+n} \ , \qquad \text{for all} \ \ m, n \in \mathbb{N} \ .$$

Prove that $\{x_n\}_{n\in\mathbb{N}}$ is an arithmetic progression.

Remarks:
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Thank you for your participation
Good Luck !

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Weekly Problem Competition

Friday, February 27, 2009

Prove or disprove: if $x, y \in \mathbb{R}$ with $y \geq 0$ and $y(y+1) \leq (x+1)^2$, then $y(y-1) \leq x^2$.

Remarks:
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Thank you for your participation
Good Luck !

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Weekly Problem Competition

Friday, March 6, 2009

Prove that there is not a strictly increasing sequence of positive integers $a_1, a_2, \ldots$ such that

$$a_{nm} = a_n + a_m \ , \qquad \text{for every } n, m \in \mathbb{N} \ .$$

Remarks:
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Thank you for your participation
Good Luck !

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Weekly Problem Competition

Friday, March 27, 2009

Let $a_1, a_2, \ldots, a_n$ be n numbers each of them being either 1 or -1 and assume that

$$a_1a_2 + a_2a_3 + \ldots a_{n-1}a_n + a_na_1 = 0\,.$$

Prove that n is divisible by 4.

Remarks:
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Weekly Problem Competition

Friday, April 03, 2009

$3n$ kids took part in a summer camp. Every day, three kids were on duty. By the end of the summer camp, any two of $3n$ kids had just one time to be on duty on the same day.

1. Prove that n is odd.
2. Is any arrangement satisfying the problem requirements for $n = 3$?

Remarks:
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Thank you for your participation
Good Luck !

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Weekly Problem Competition

Friday, April 10, 2009

The numbers 1, 2, 3, 4, 5, 6, 7, 8 are placed at the vertices of a given cube (one number at one vertex). Assume that the sum of any three numbers on any face is not less than 10. Denote by S_k, $k = 1, 2, 3, 4, 5, 6$, the sum of the four numbers on the k-th face. Among all cubes, find the smallest possible value of $\min_k S_k$.

Remarks:
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Thank you for your participation
Good Luck !